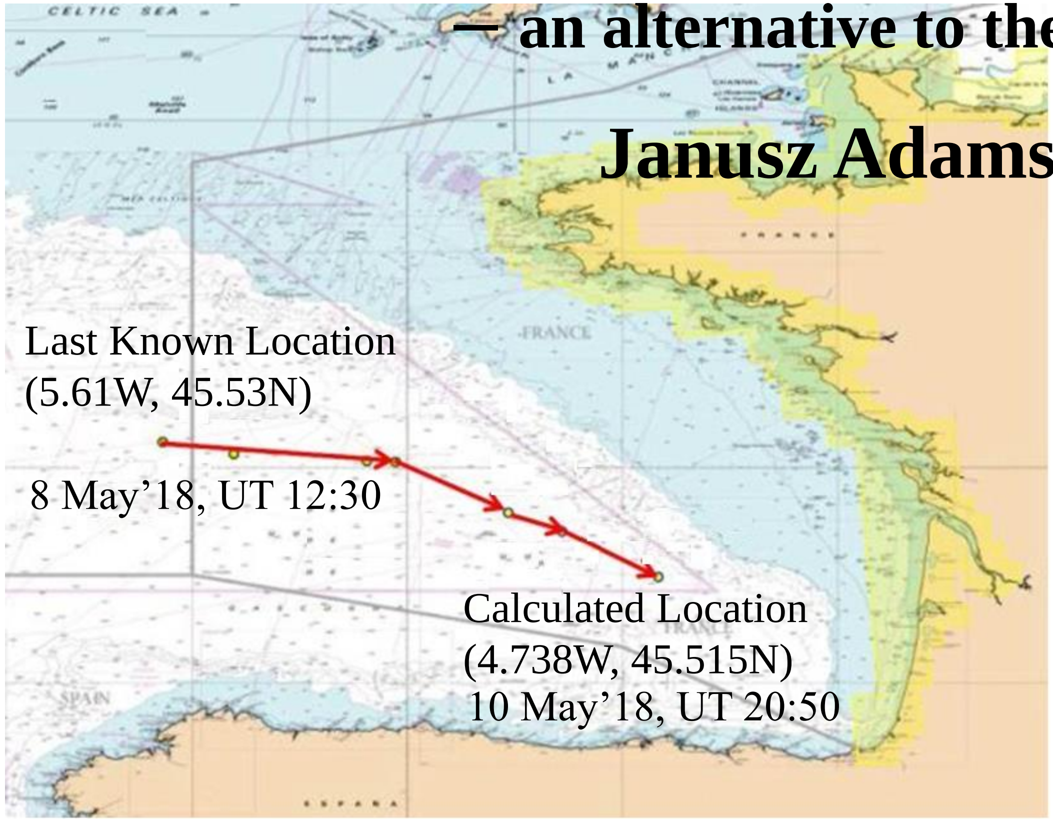
The use of Pulsars for Ship Navigation

– an alternative to the sextant

Janusz Adamson



Scenario

- Severe storm in Bay of Biscay
- Rescue operations suspended
- Total power failure
- Natural, human interference
  - Communications loss
  - No GNSS/GPS signal
  - No radar
- Cloudy, sextant useless

Battery power available - for essential functions

Ship’s position estimated using Dead Reckoning

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Content

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- ❑ NASA & ESA X-ray navigation experiments
- ❑ Radio Interferometry
- ❑ Assumptions & Initial Conditions
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- ❑ Summary

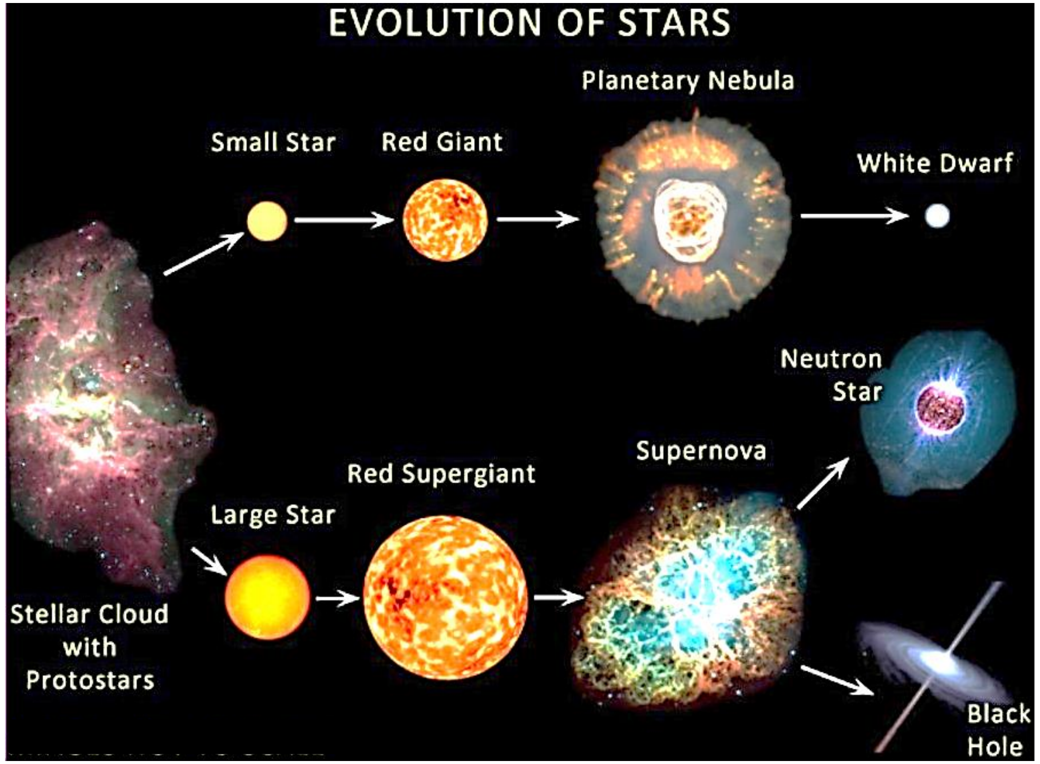
Stellar Nuclear Reactions – basic process

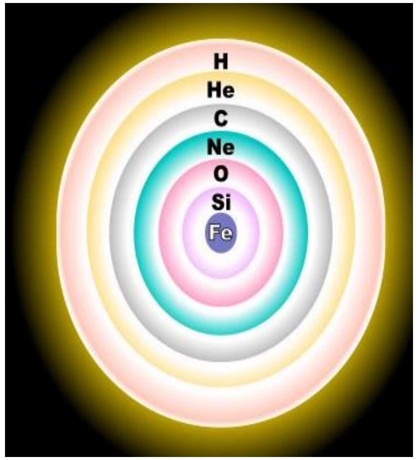
$4H^1 \rightarrow He^4$
↓
C^{12}
↓
O^{16}
↓
Ne^{20}
↓
Mg^{24}
↓
Fe
↓
Heavy elements

(hydrogen burning is main stellar process)

exothermic reactions (energy output) — He^4 through Mg^{24}

endothermic reactions (energy input) — Mg^{24} through heavy elements

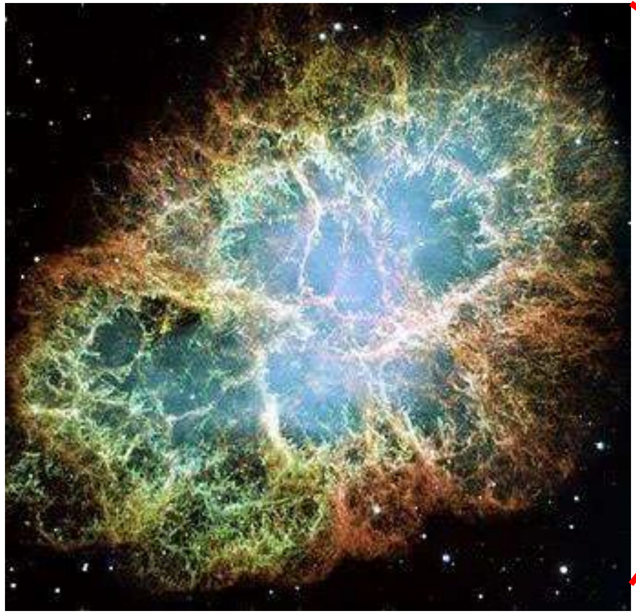


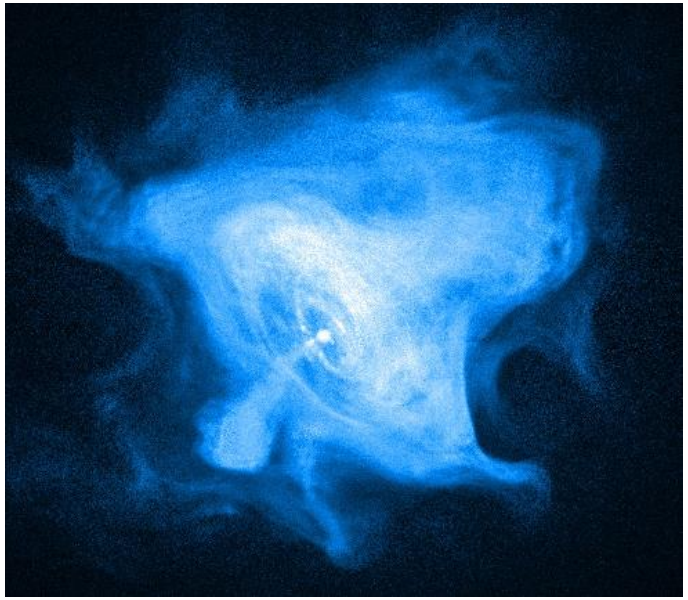


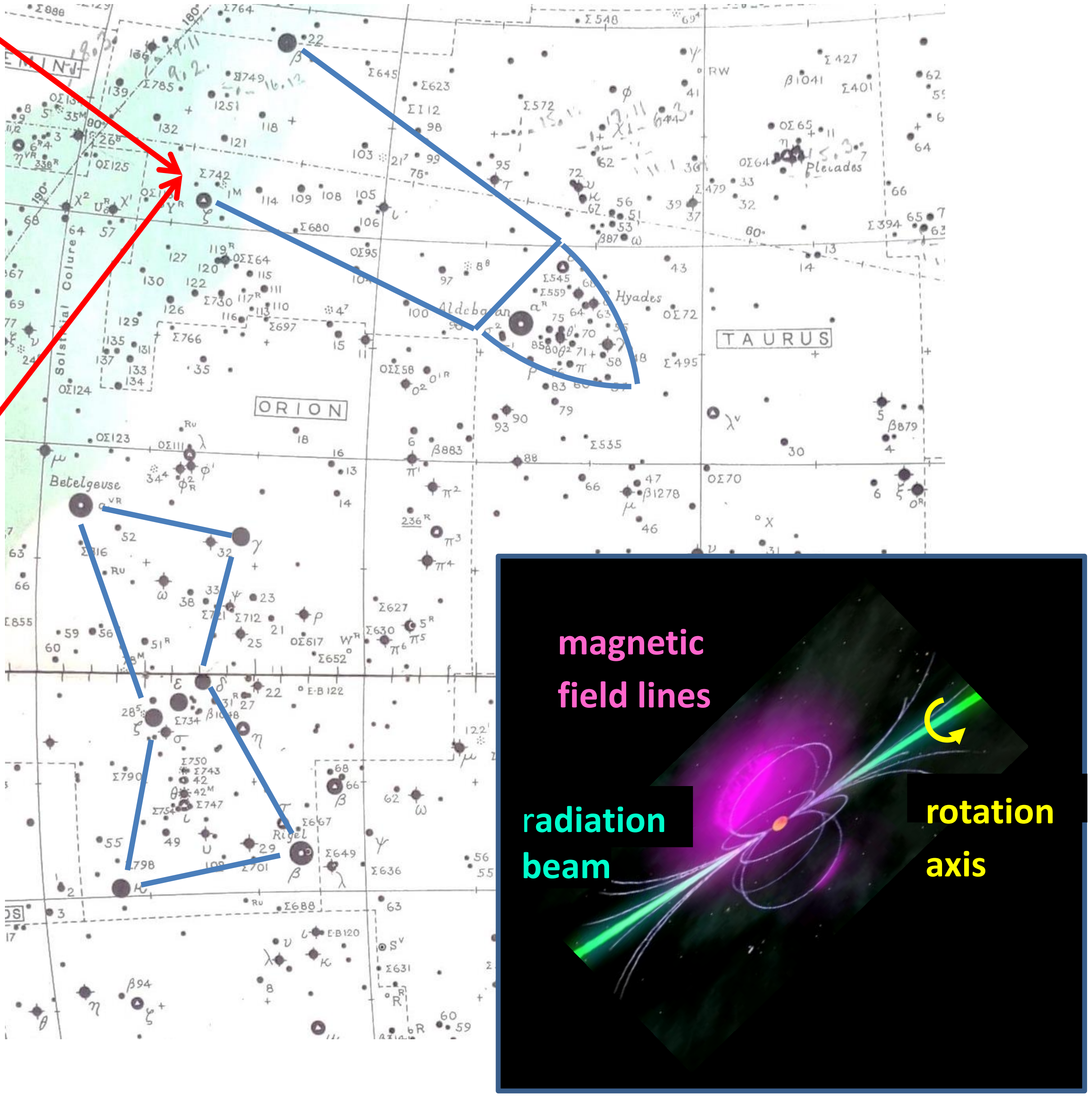
Star Type	Mass
White Dwarf	< 1.4 Solar Mass
Neutron Star	> 1.4 Solar Mass < 3.8
Black Hole	> 3.8 Solar Mass

Crab Nebula Pulsar – Best known Pulsar

(rotating neutron star)

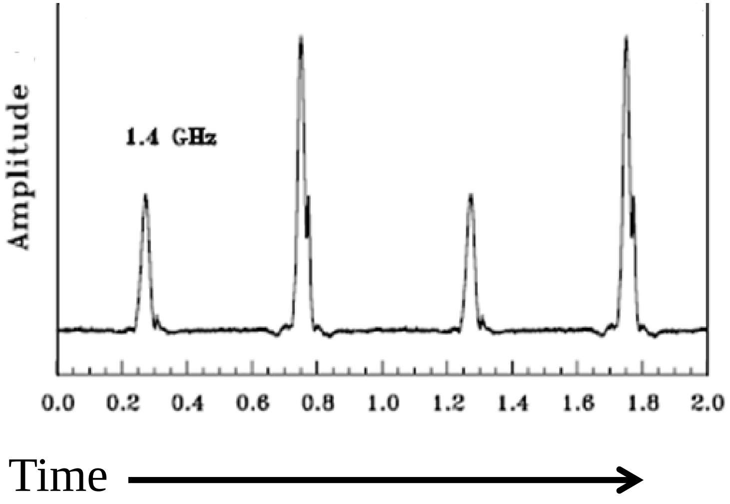




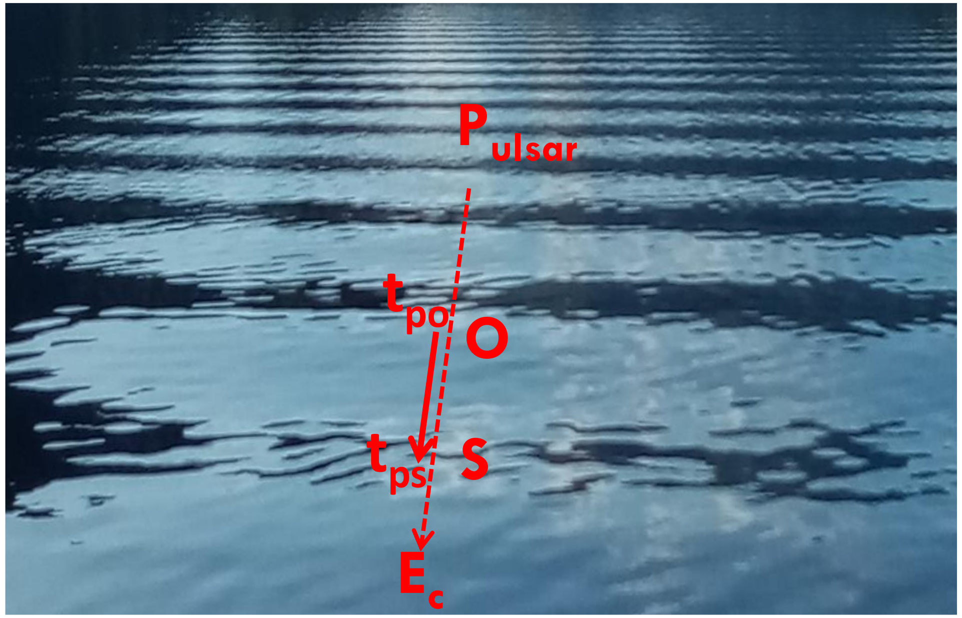


Use of Pulsars for ship navigation

Arrival time of pulses can be predicted to a very high precision because of their long term timing stability (~ 1 in 10^{13})



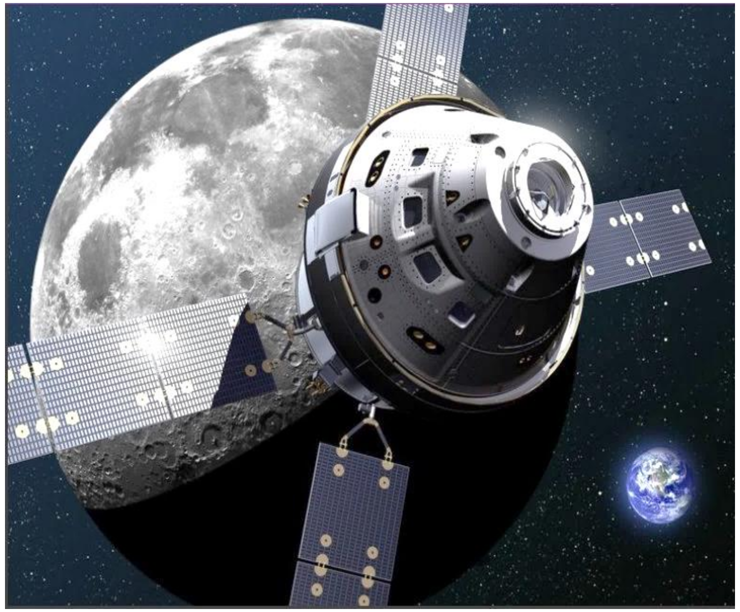
Periodic pulses over many Pulsar rotation cycles



distance = ($t_{ps} - t_{po}$) x c

NASA X-ray Navigation Experiment

NICER (Neutron Star Interior Composition Explorer)/ SEXTANT (Station Explorer for X-ray Timing and Navigation Technology) experiment on the International Space Station (ISS)



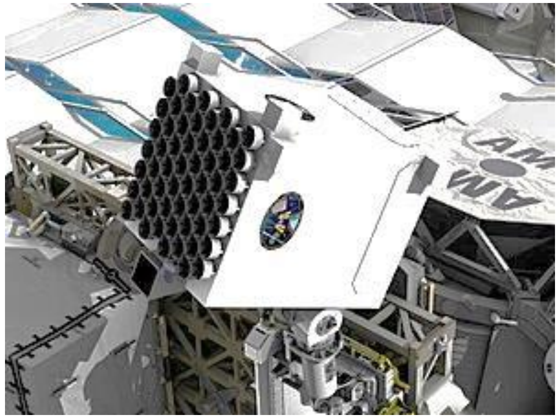
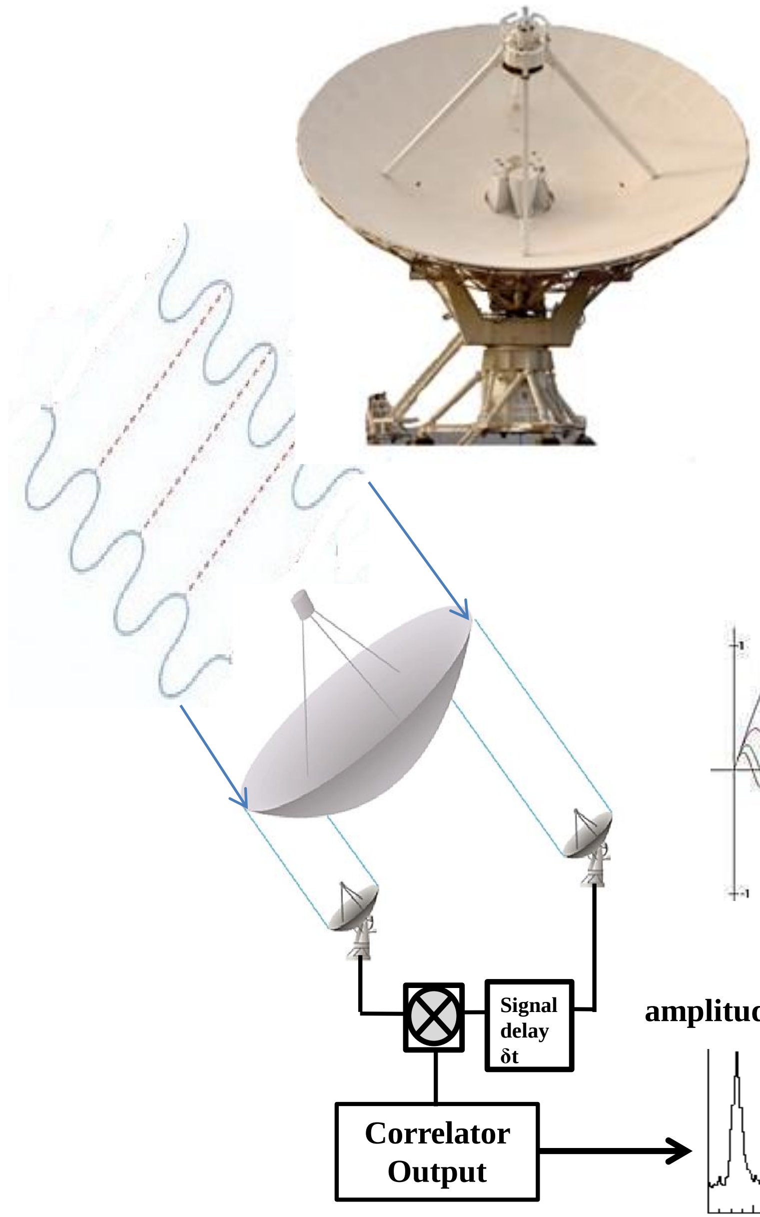
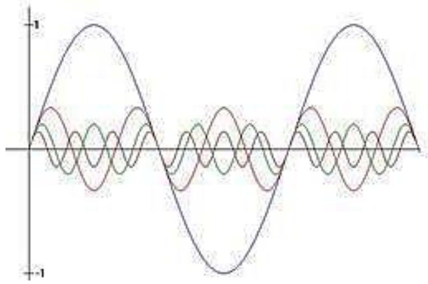


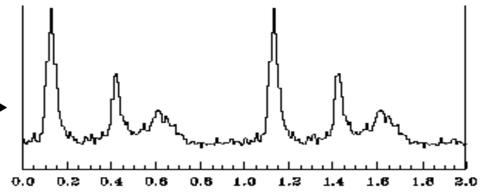
Illustration of NICER phased-array antenna aboard the ISS

Orion manned spacecraft

Radio telescopes – radio interferometry







Parkes 65m antenna too large, heavy, cumbersome

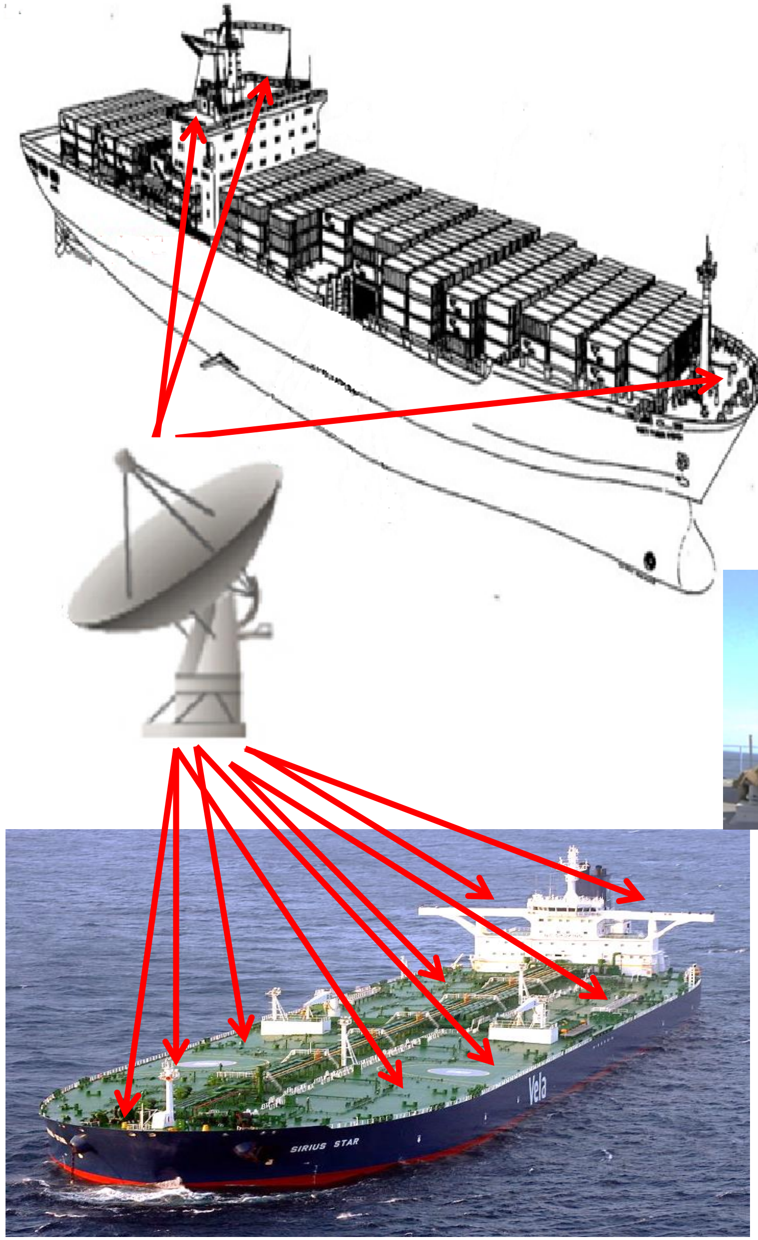
Radio interferometric approach

Virtual antenna with area equal to size of cluster

Signals are amplified, digitized at the antennas and then integrated from a cluster of radio telescopes in the correlator.

Signal integration required before Pulsar pulse timing difference measurements can begin

Pulsar-Based Ship Navigation – potential superstructure locations for 3 or 9 x 10m^2 radio antennas??



Signal integration time for Parkes radio telescope 60s - 120s. Timing measurement accuracy 1 in 10^{13}s

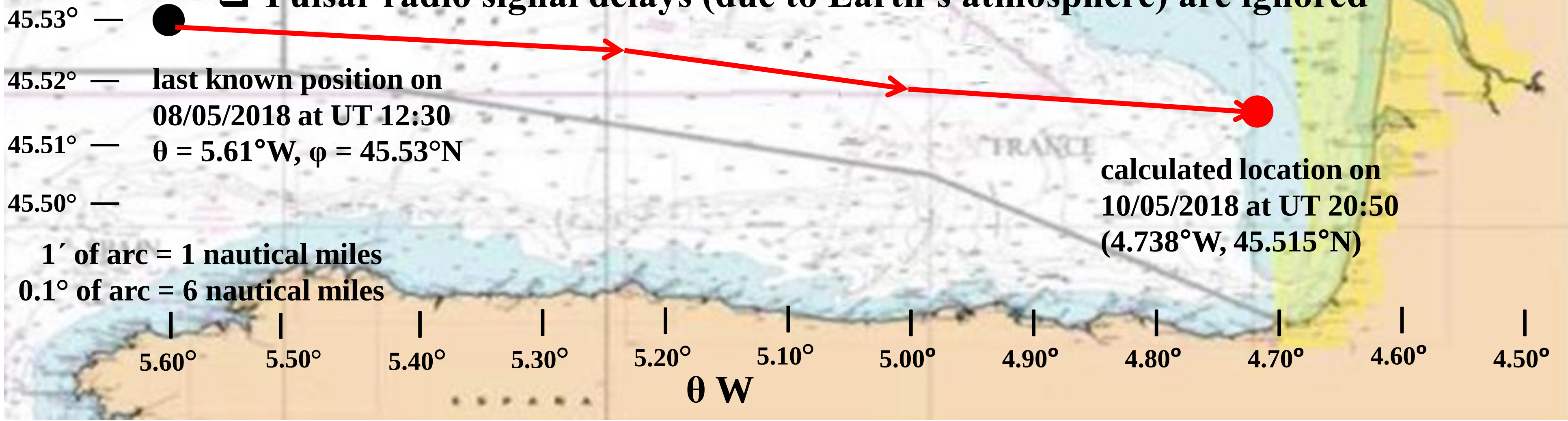
(Antenna area)^2 α (integration time)$^{-1}$

10^{2}m radio antenna signal integration time $\geq$2min. Timing measurement accuracy 1 in 10^{6}s required

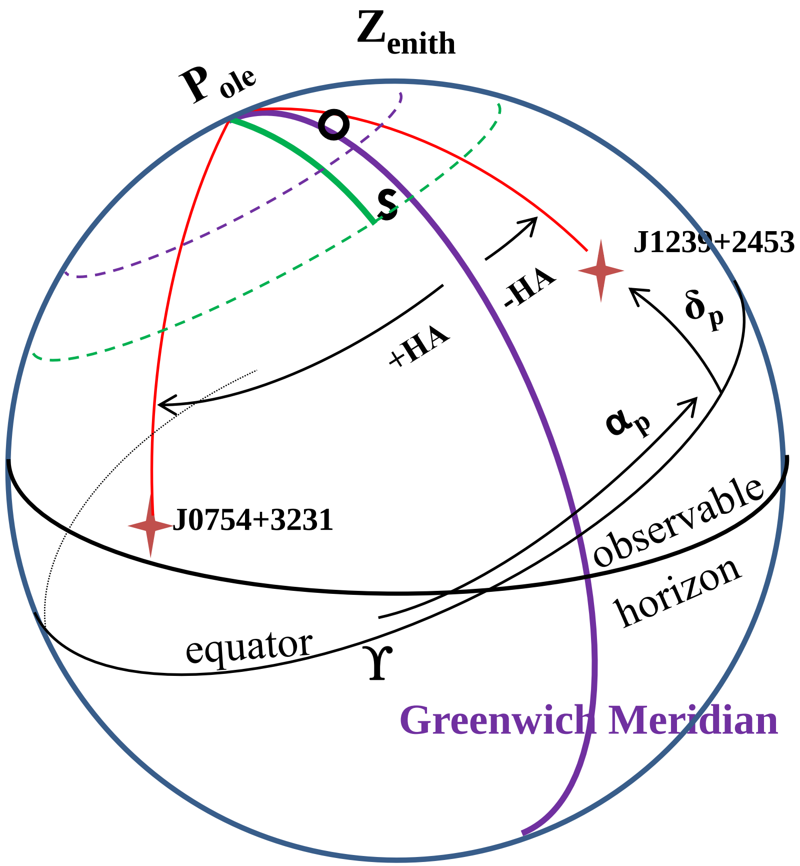
Antenna tracking based on Type 45 Destroyer naval gun motion and positioning system in all sea conditions

Assumptions & Initial Conditions

- ❑ Pulse timing measurement accuracy of 10^{-6} s (to match NASA X-ray experiments) equates to a distance error of ±1km
- ❑ Earth is a geoid/ellipsoid, but assumed a spheroid at specified locations (e.g. ship's location) during pulse timing measurements; angular error ≈1.2″arc
- ❑ Geocentric radius rounded to 1km
- ❑ Ship stationary during pulse timing measurements, drift error ≈ 240m
- ❑ Unknown r_s value assumed as last known ship's value, error < 1m
- ❑ GMST ≈ GAST = GST position error ≈30m
- ❑ Greenwich chosen as reference location instead of Jodrell Bank
- ❑ Pulsar radio signal delays (due to Earth's atmosphere) are ignored



Spherical Astronomy – basic parameters



(θ_s, φ_s) = Ship's longitude & latitude

HA_p = Hour Angle of Pulsar

G.S.T. = Greenwich Sidereal Time

G.S.T. = $HA_p + RA_p$

α_p = Pulsar's Right Ascension (RA_p)

δ_p = Pulsar's Declination

♈ = First Point of Aries (equinox)

G.S.T. = GMST + equ. of equinoxes = GAST

r = geocentric radius

ε = 1/298.257

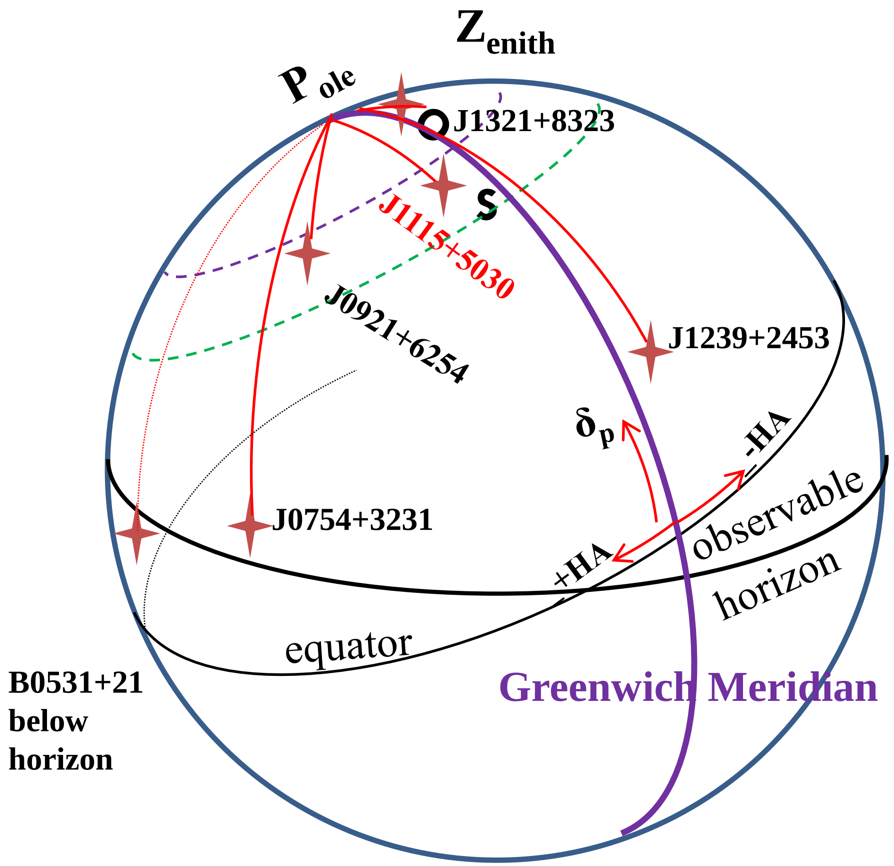
Sign convention

Longitude east, Greenwich least (i.e. –ve)

Longitude west, Greenwich best (i.e. +ve)

Pulsar locations

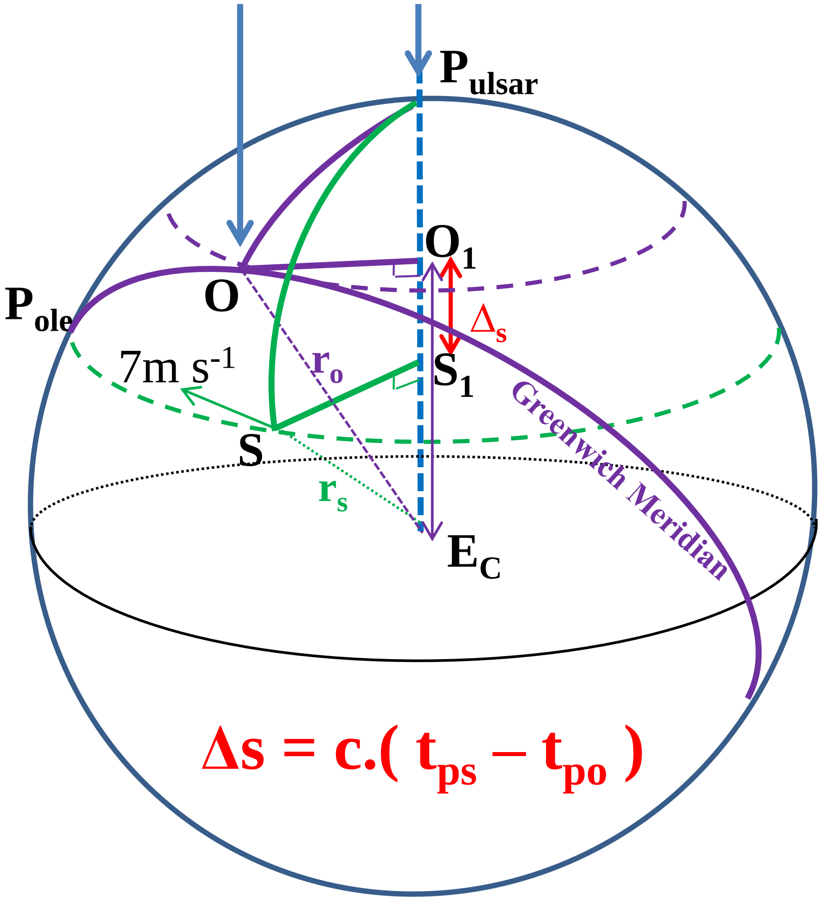
– selected for illustration purposes only



Date & Time	10th May 2018	UT 20:50
Pulsar ID J2000 notation	Hour Angle (HA) From GM	Dec (δp) (degrees)
J0754+3231	62.43396°	32.53228°
J1115+5030	12.19350°	50.50341°
J0921+6254	40.79460°	62.90387°
J1239+2453	-8.81509°	24.89702°
J1321+8323	-19.33892°	83.39414°
Crab Nebula Pulsar		
B0531+21	97.47029°	22.01447°

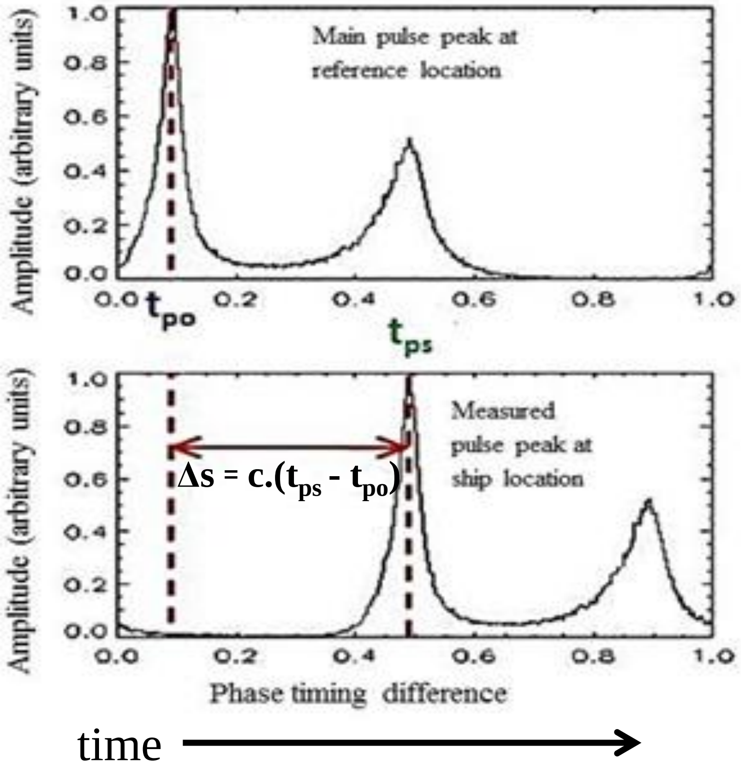
3 Pulsars = unique solution
4+ Pulsars = over-determined solution

Pulsar – Pulse Peak Timing Difference Measurements



O_1 & S_1 are projections of reference observatory (O) and unknown ship position (S) onto the Pulsar-Earth centre line

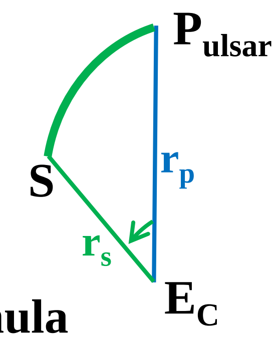
Date	10th May 2018	UT 20:50
J2000notation	Timing difference	Distance Δs
J0754+3231	≈7.61 x 10^{-4} s	-228 ± 1km
J1115+5030	≈4.0 x 10^{-5} s	-12 ± 1km
J0921+6254	≈3.77 x 10^{-4} s	113 ± 1km
J1239+2453	≈6.57 x 10^{-4} s	-197 ± 1km
J1321+8323	≈1.321 x 10^{-3} s	396 ± 1km

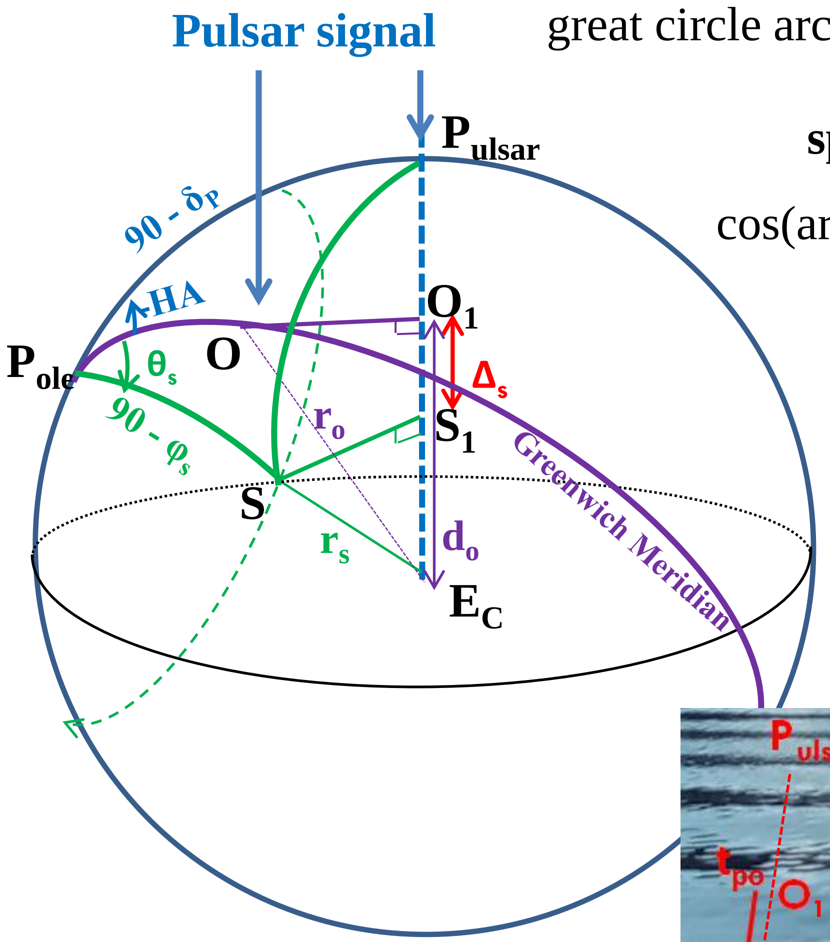


Spherical Astronomy – properties & equations

1′ of arc = 1 nautical mile
0.1° of arc = 6 nautical miles

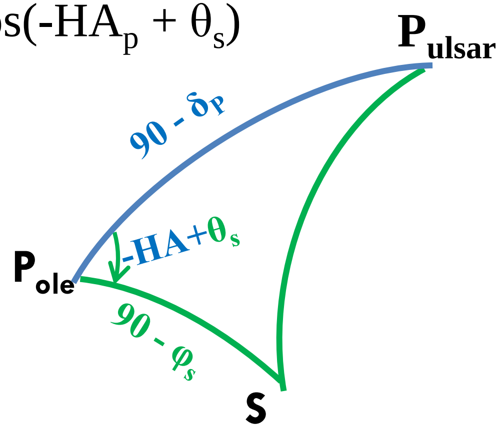
great circle arc(SP_{ulsar}) = angle(SE_CP_{ulsar})





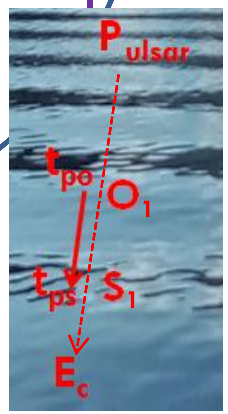
spherical trigonometric cosine formula

$$\cos(\text{arc}SP_{ulsar}) = \cos(90 - \varphi_s)\cos(90 - \delta_p) - \sin(90 - \varphi_s)\sin(90 - \delta_p)\cos(-HA_p + \theta_s)$$



basic trigonometric function

$$d_o = r_o\cos(OE_CP_{ulsar})$$
$$d_s = r_s\cos(SE_CP_{ulsar})$$
$$r \approx 6378(1 - \varepsilon\sin^2\varphi)\ \text{km}$$

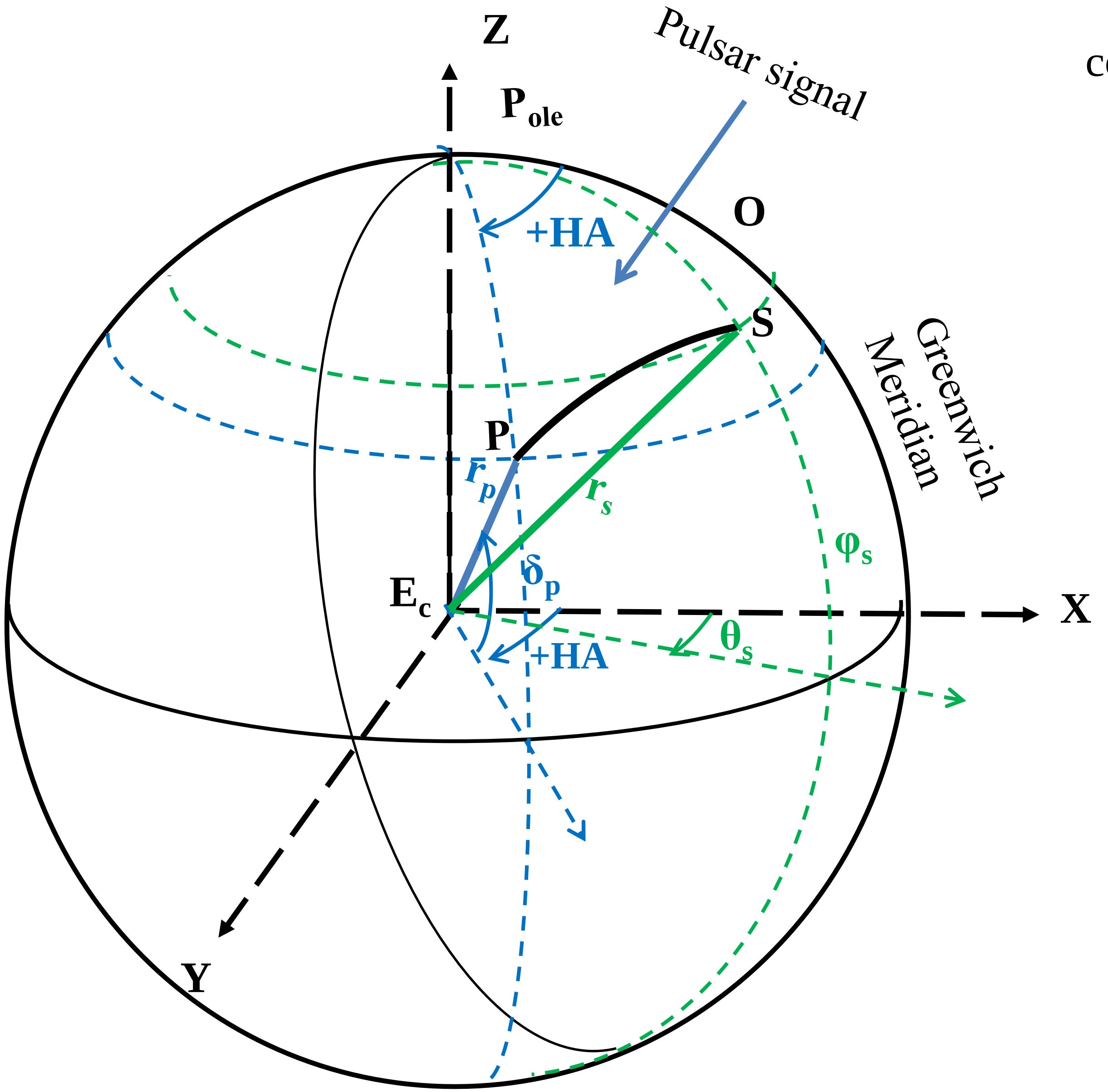


$$d_o = \overrightarrow{E_CO_1}$$
$$d_s = \overrightarrow{E_CS_1}$$
$$d_s = d_o - \Delta_s$$
$$\Delta s = (t_{ps} - t_{po}) \times c$$

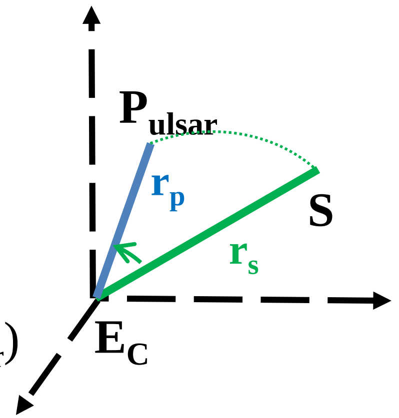
Sign convention

Longitude east	–ve
Longitude west	+ve
H.A. east	–ve
H.A. west	+ve

Vector scalar product



$$\cos(SE_C P_{ulsar}) = \frac{\mathbf{r_s} \cdot \mathbf{r_p}}{|\mathbf{r_s}|\,|\mathbf{r_p}|}$$



and $\quad d_s = r_s \cos(SE_C P_{ulsar})$

so $\quad |\mathbf{r_p}|\, d_s = \mathbf{r_s} \cdot \mathbf{r_p}$

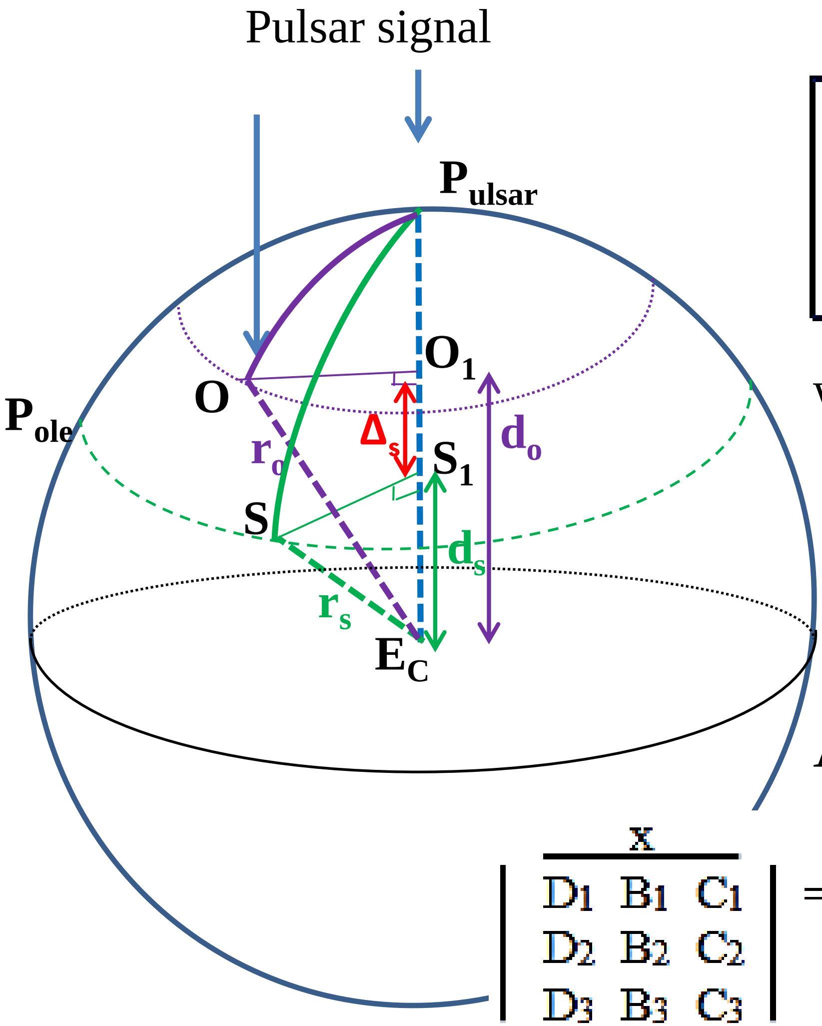
$$= x_s . x_p + y_s . y_p + x_s . z_p$$

coordinate transformation

$$x_s = r_s \cos(\varphi_s)\cos(\theta_s)$$
$$y_s = r_s \cos(\varphi_s)\sin(\theta_s)$$
$$z_s = r_s \sin(\varphi_s)$$

$$x_p = r_p \cos(\delta_p)\cos(HA_p)$$
$$y_p = r_p \cos(\delta_p)\sin(HA_p)$$
$$x_p = r_p \sin(\delta_p)$$

Solution – 4 signal timing measurements



Solutions in matrix form are written as

$$\begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \\ A_4 & B_4 & C_4 \end{bmatrix} \begin{bmatrix} x \\ yf \\ yg \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

where

A, B ,C, D are calculated values

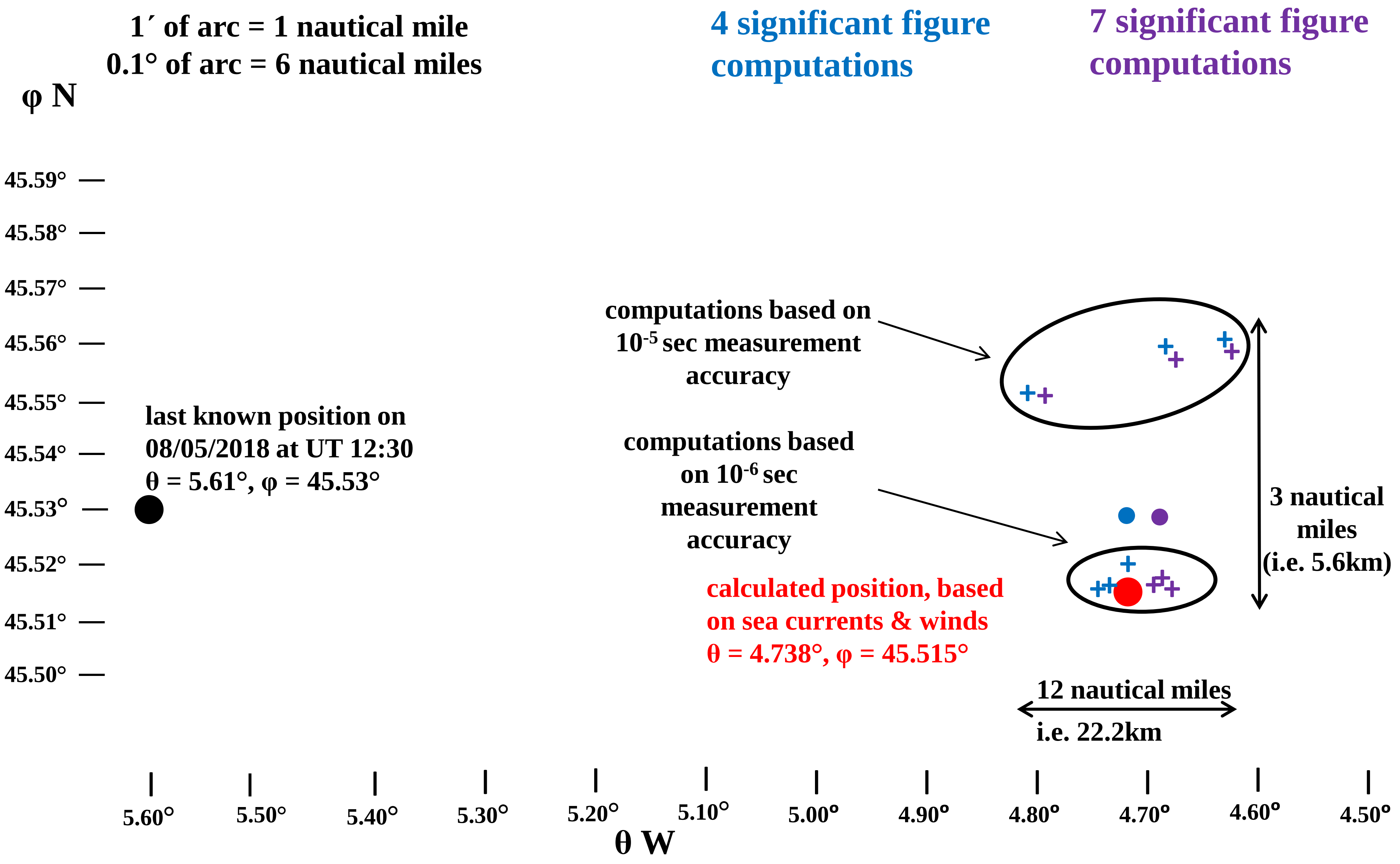
x, y, f, g are unknown functions for (θ_s, φ_s)

change in r_s is minimal, hence ignored

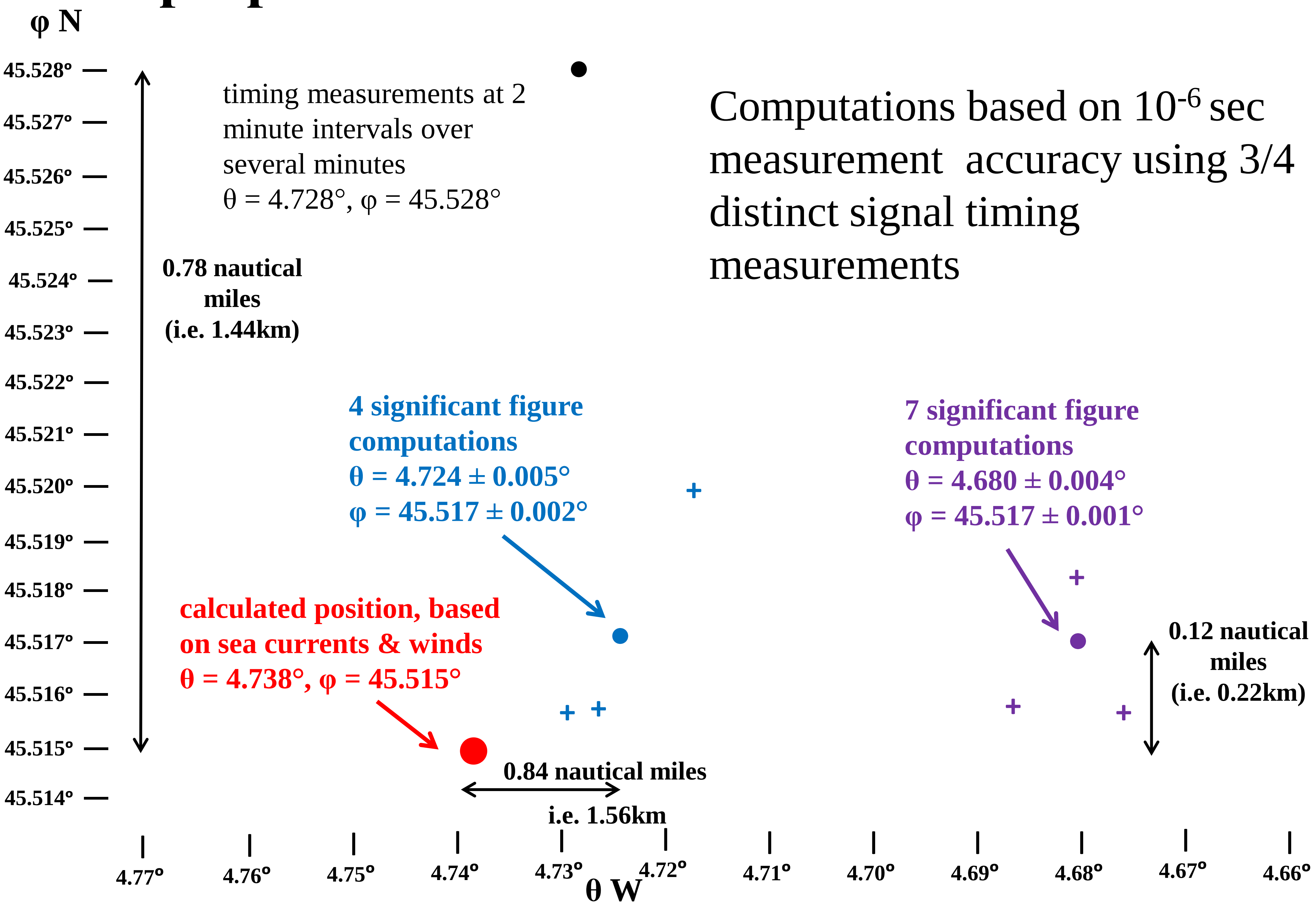
A solution for x, y, f, g, can be written as

$$\frac{x}{\begin{vmatrix} D_1 & B_1 & C_1 \\ D_2 & B_2 & C_2 \\ D_3 & B_3 & C_3 \end{vmatrix}} = \frac{yf}{\begin{vmatrix} A_1 & D_1 & C_1 \\ A_2 & D_2 & C_2 \\ A_3 & D_3 & C_3 \end{vmatrix}} = \frac{yg}{\begin{vmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \\ A_3 & B_3 & D_3 \end{vmatrix}} = \frac{1}{\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}}$$

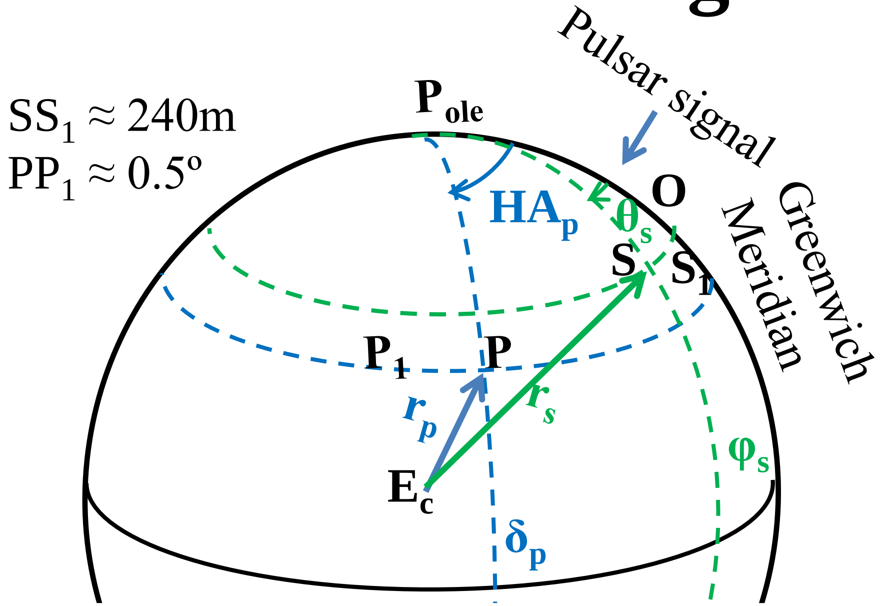
Ship's position on 10/05/2018 at UT 20:50



Ship's position on 10/05/2018 at UT 20:50



Single Pulsar-Based Ship Navigation

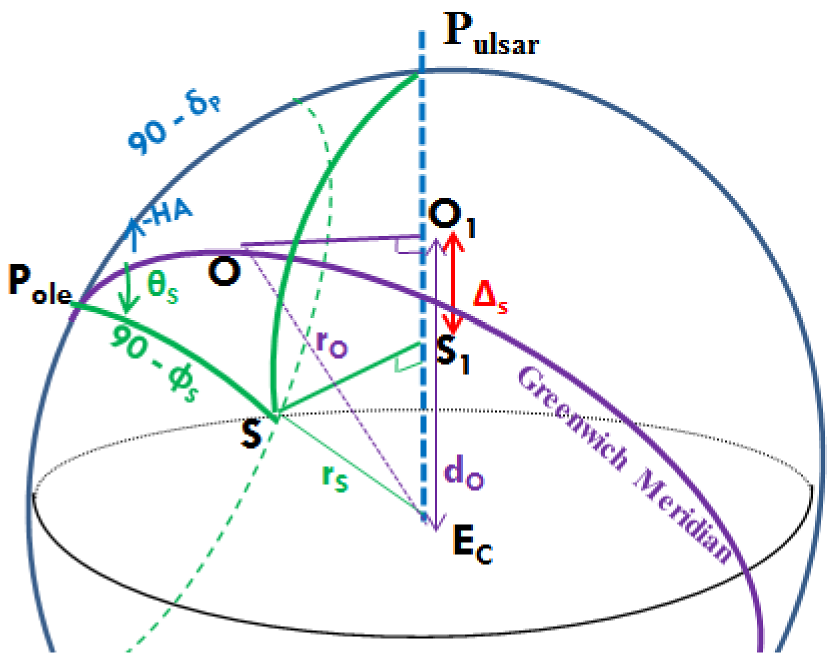


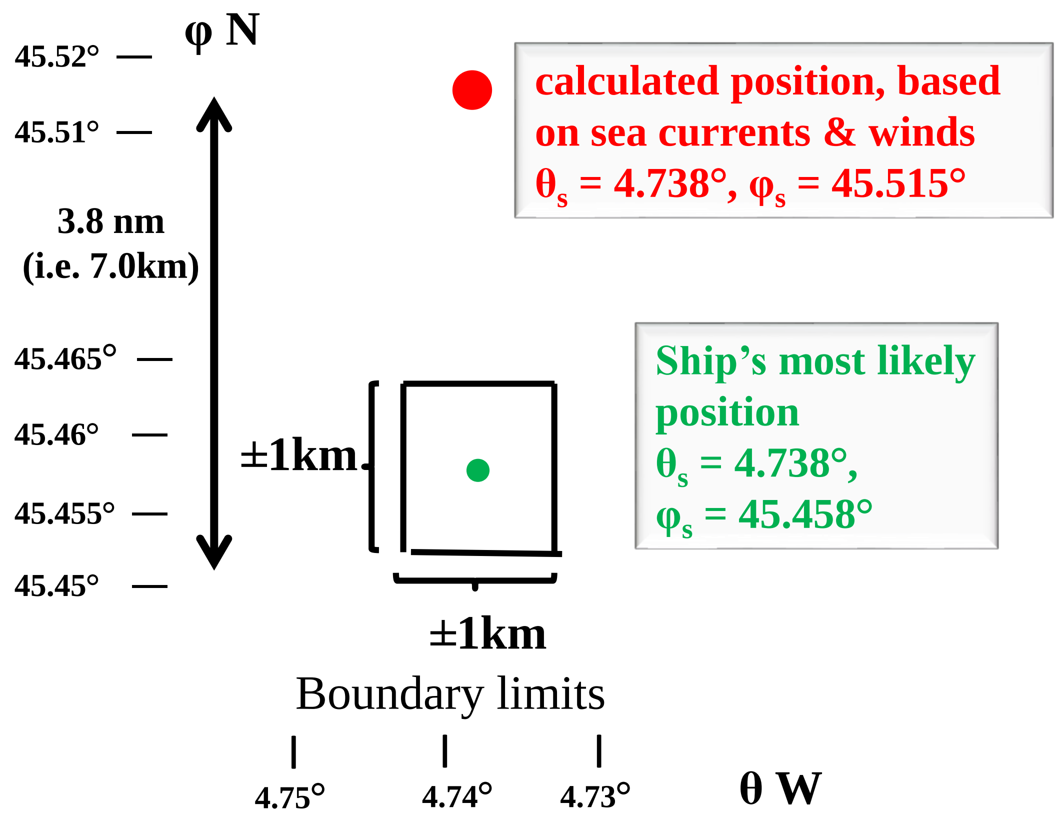
Available   last known ship's position

θ_s = 5.61°W, φ_s = 45.53°N,

Pulsar values α_p, δ_p, HA_p and Δs at 2 minute intervals

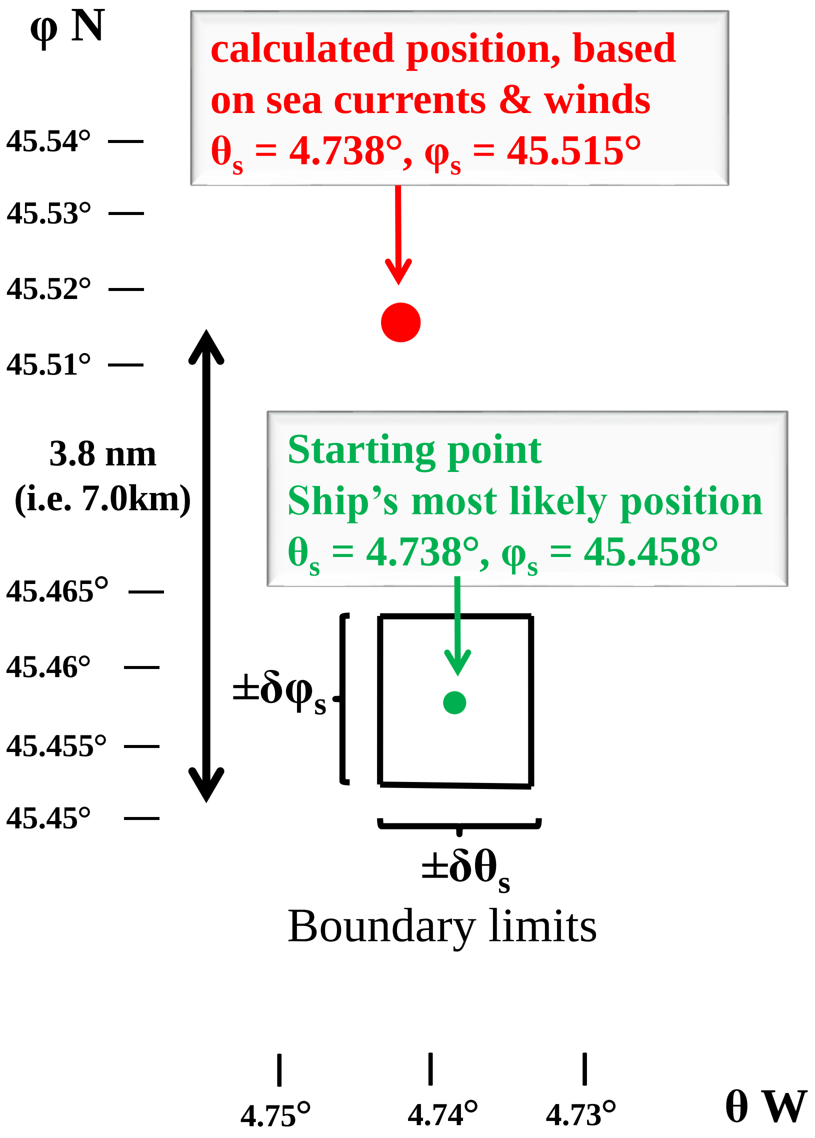
Calculations using vector product & cosine formula give ship's most likely position θ_s = 4.738°, φ_s = 45.458°

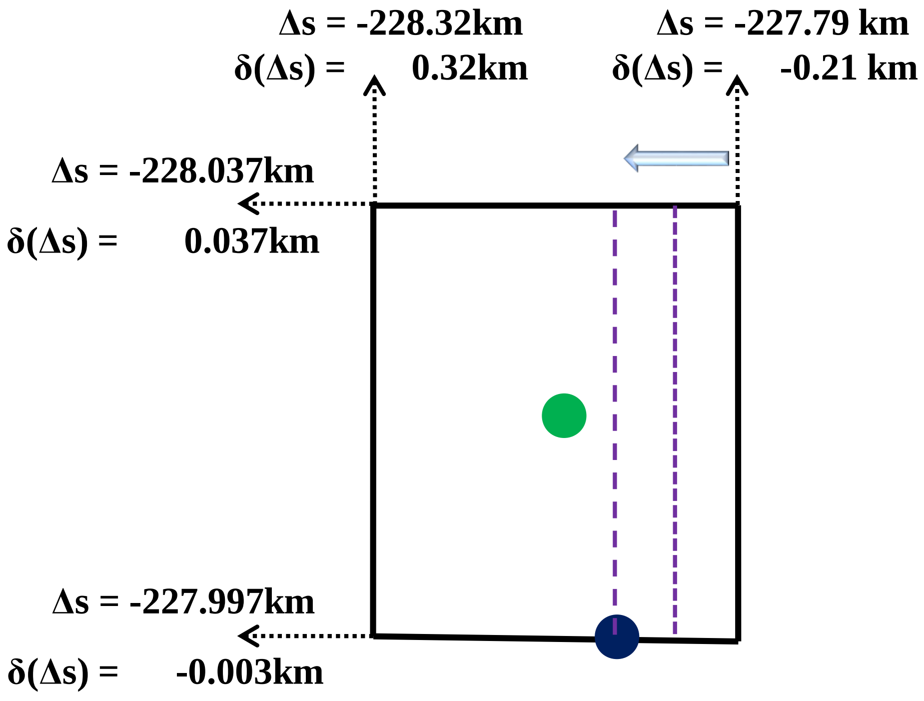




Incremental Change – using Pulsar J0754+3231

Method use Taylor's Theorem to calculate $\delta\theta$, $\delta\varphi$ followed by Divide & Conquer

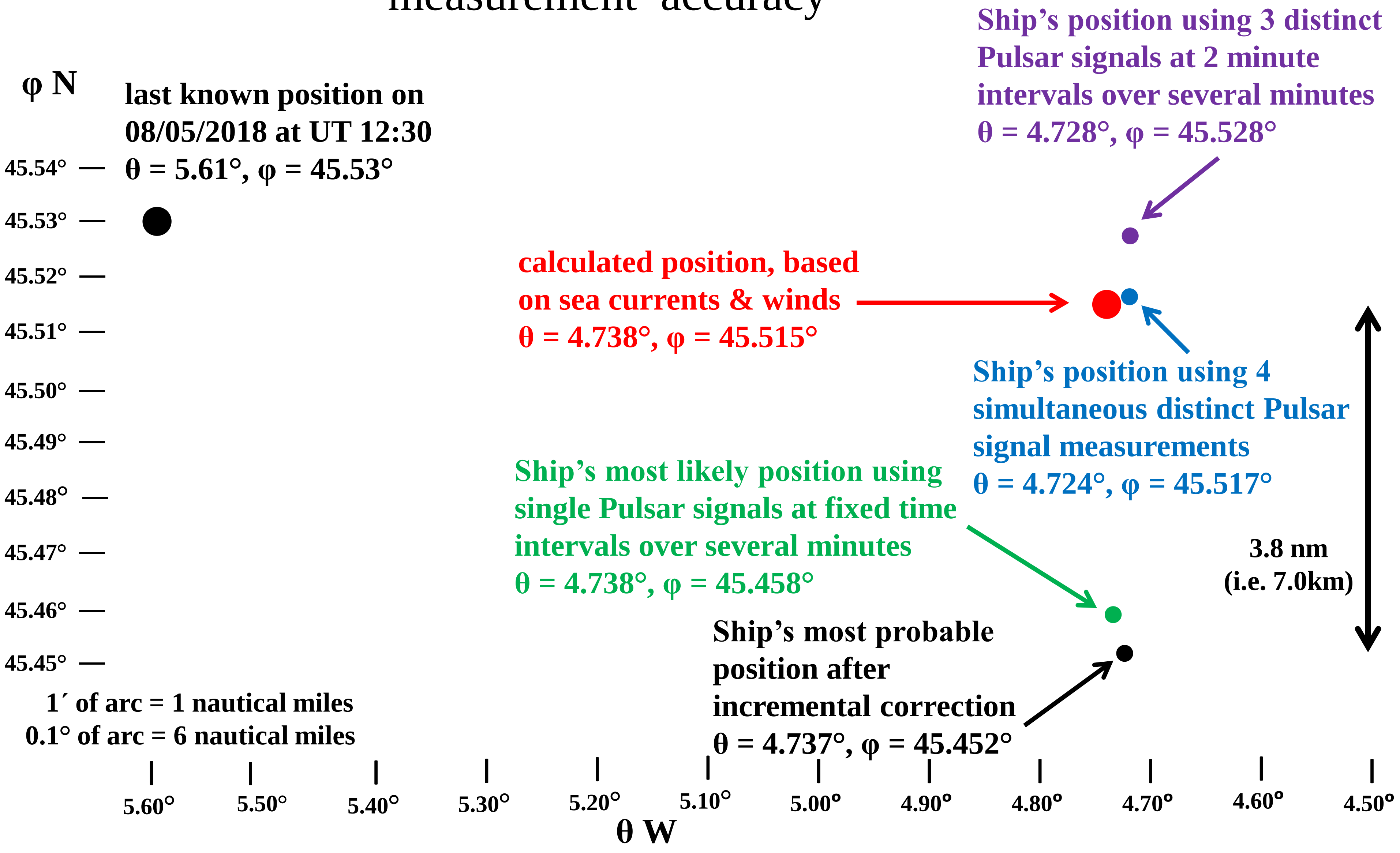




Ship's most probable position
$\theta_s = 4.737°$, $\varphi_s = 45.452°$

Ship’s position on 10/05/2018 at UT 20:50

Computations based on 10^{-6} sec measurement accuracy



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– an alternative to the sextant

Summary

- Earth's atmosphere opaque to X-rays.
- An interferometric approach using a cluster of small radio antennas, to be located on the ship's superstructure.
- Position accuracy achieved to within 2km.
- Single Pulsar signal calculations provide a ship's position to within 7km.
- 10^{-6} pulse profile resolutions to be comparable with NASA's and ESA's spacecraft navigation experiments.
- Order of magnitude position improvement could be achieved using 10^{-7} pulse profile measurement resolution.
- Challenge – provide improved position accuracies in the next few years.