Modelling Quantum Sensors in a Realistic Navigation System Context

ESA NAVISP EL1-089 MOQUA

Final Presentation



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11/11/26

Outline

Background and Motivation

Objectives of project

Project Work Packages

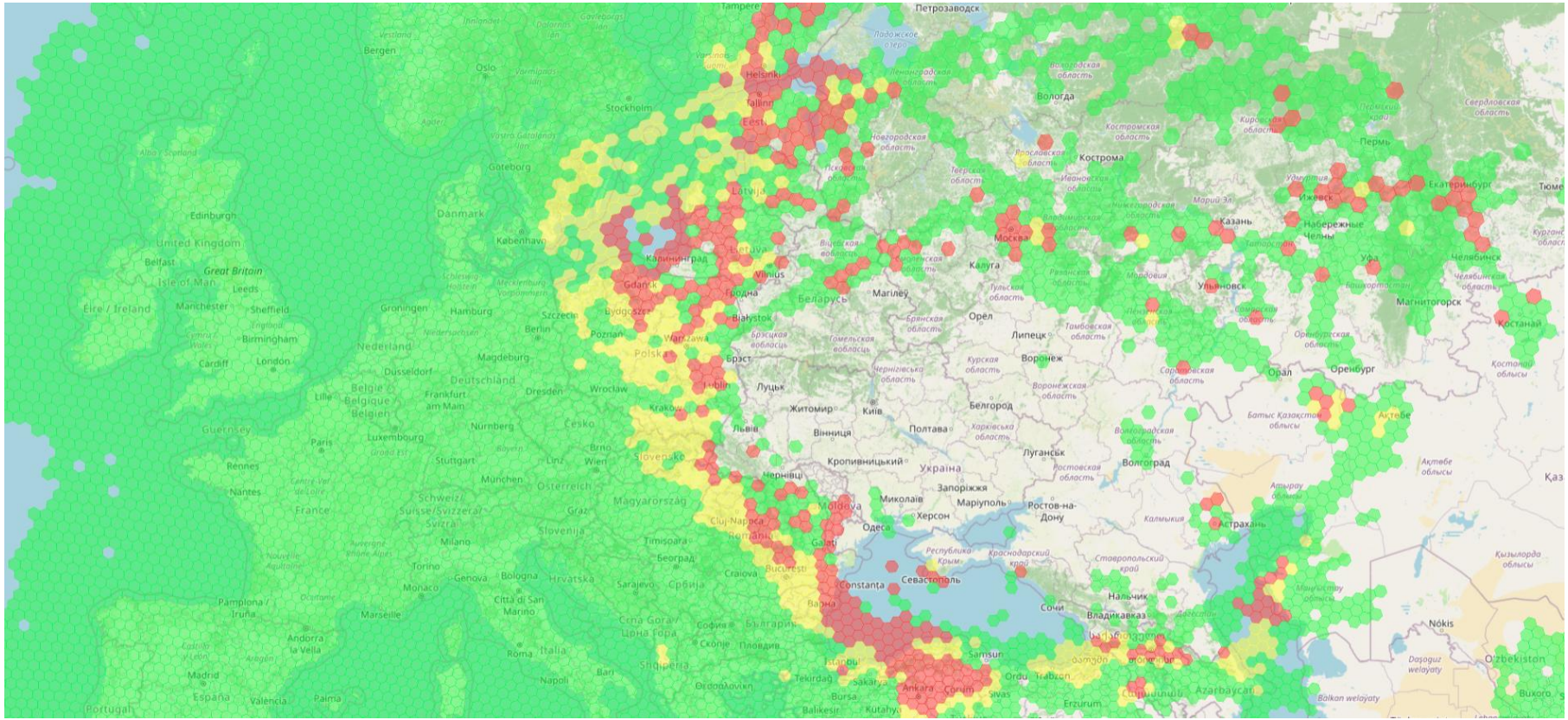
Conclusions and Recommendations

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Background and Motivation

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GNSS is vulnerable



From https://gpsjam.org/

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Could quantum sensors help?

Lots of quantum sensors to choose from!

- Ranging/LiDAR sensors
- Magnetometers
- Gravimeters/gradiometers
- **Inertial sensors (QIS) based on cold atom interferometry (CAI)**
- Clocks



Field trials of CAIs

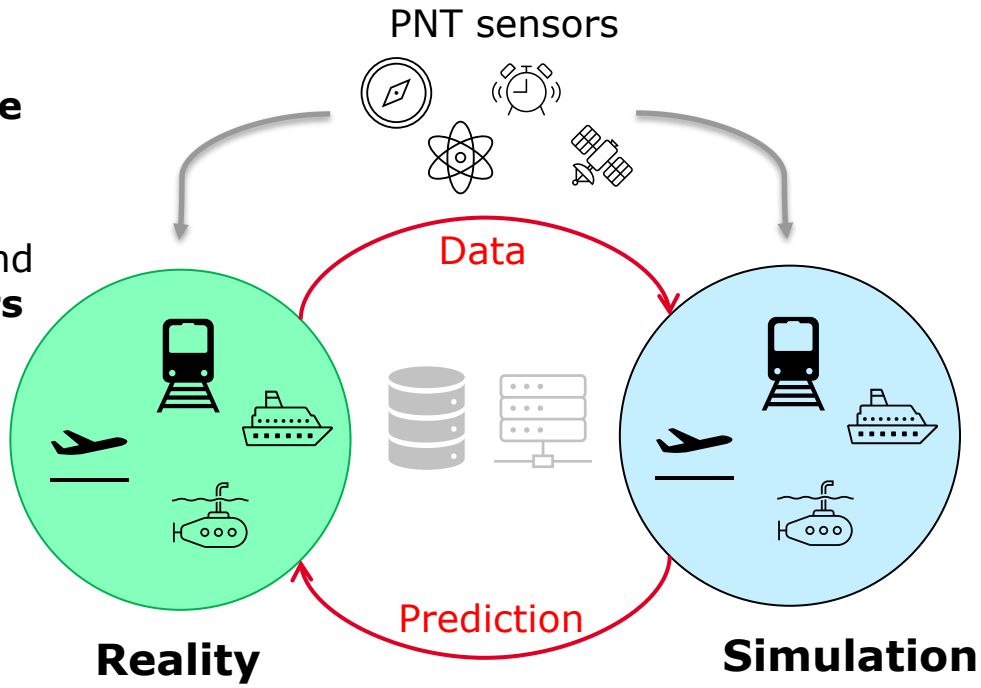
- Numerous field trials undertaken by various groups: land, sea, air
- Proof-of-concept measurements obtained – but this does not guarantee good PNT
- Challenges remain

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Image: https://www.gov.uk/government/news/un-jammable-quantum-tech-takes-flight-to-boost-uks-resilience-against-hostile-actors

Modelling and Simulation helps aid PNT Resilience

- Allows PNT system designers to **analyse the effect of threats** and **new technologies**
- Facilitates extensive, repeatable trade-off and investigation into **key performance drivers**
- Allows isolation of the effects of individual error sources
- Navigation/fusion algorithm trade-off
- Enables evaluation of sensors' performance and **integration potential** into PNT systems



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Objectives of MOQUA project

ESA NAVISP EL1-089

1. Generate **representative measurements** from quantum sensor model
2. Include **realistic PNT** test scenarios
3. PNT fusion engine incorporating quantum sensor measurements
4. Perform system experimentation and trade-off
5. Evaluate quantum sensors' performance and **integration potential** in PNT systems: *do they help reduce PNT risk?*



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Project Structure



***GMV: Prime**. Modelling, simulation and sensor fusion expertise*
***UoB: Subco**. Quantum sensing, modelling and fusion expertise*

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WP1000: State of the Art Review and Technical Background

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WP1000

State of the Art Review: Cold Atom Interferometry (CAI)

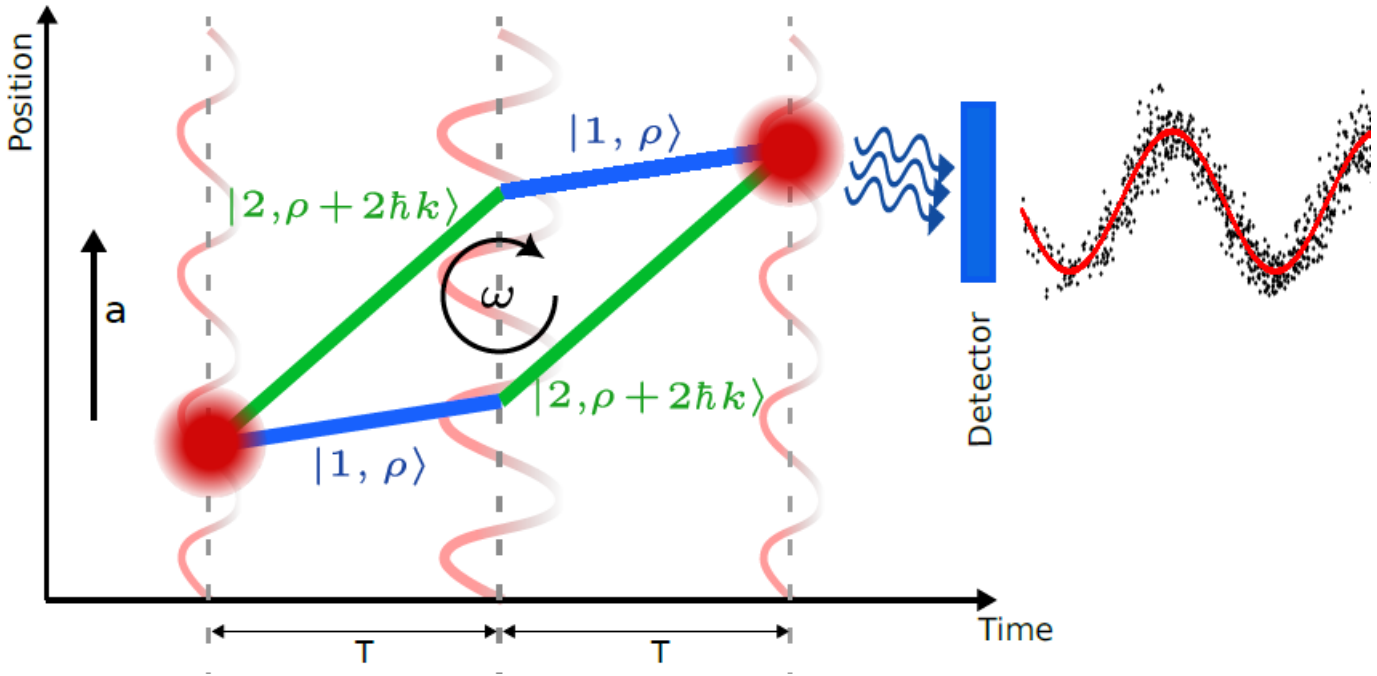
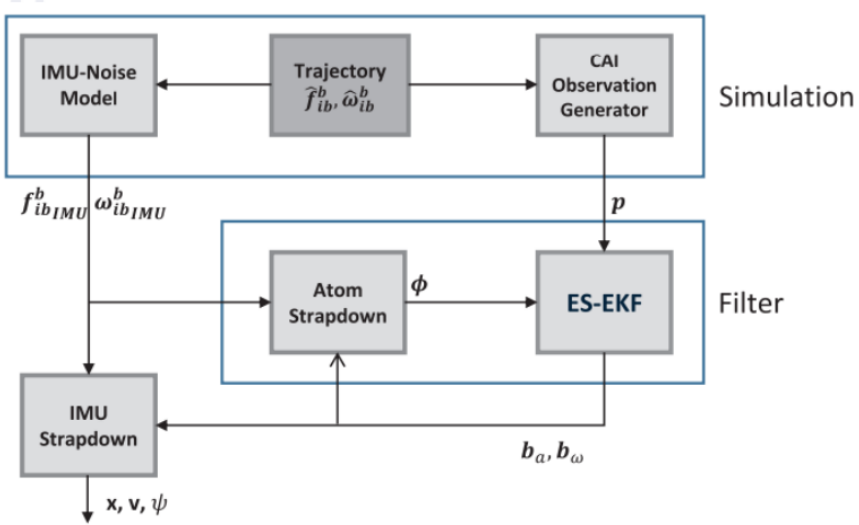


Diagram from Tennstedt, "Concept and Evaluation of a Hybridization Scheme for Atom Interferometers and Inertial Measurement Units," PhD thesis, 2025

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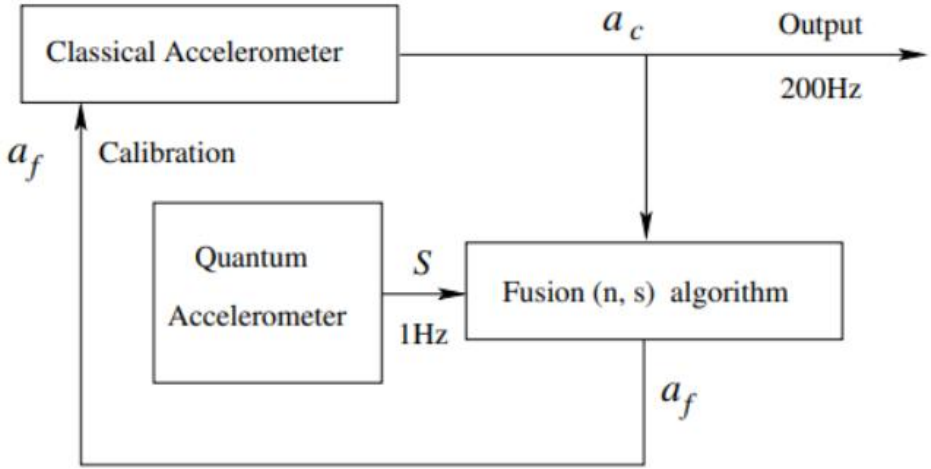
WP1000

State of the Art Review: PNT Fusion Techniques



CAI domain fusion

Tennstedt et al., "Atom Strapdown: Toward Integrated Quantum Inertial Navigation Systems", 2023



IMU domain fusion

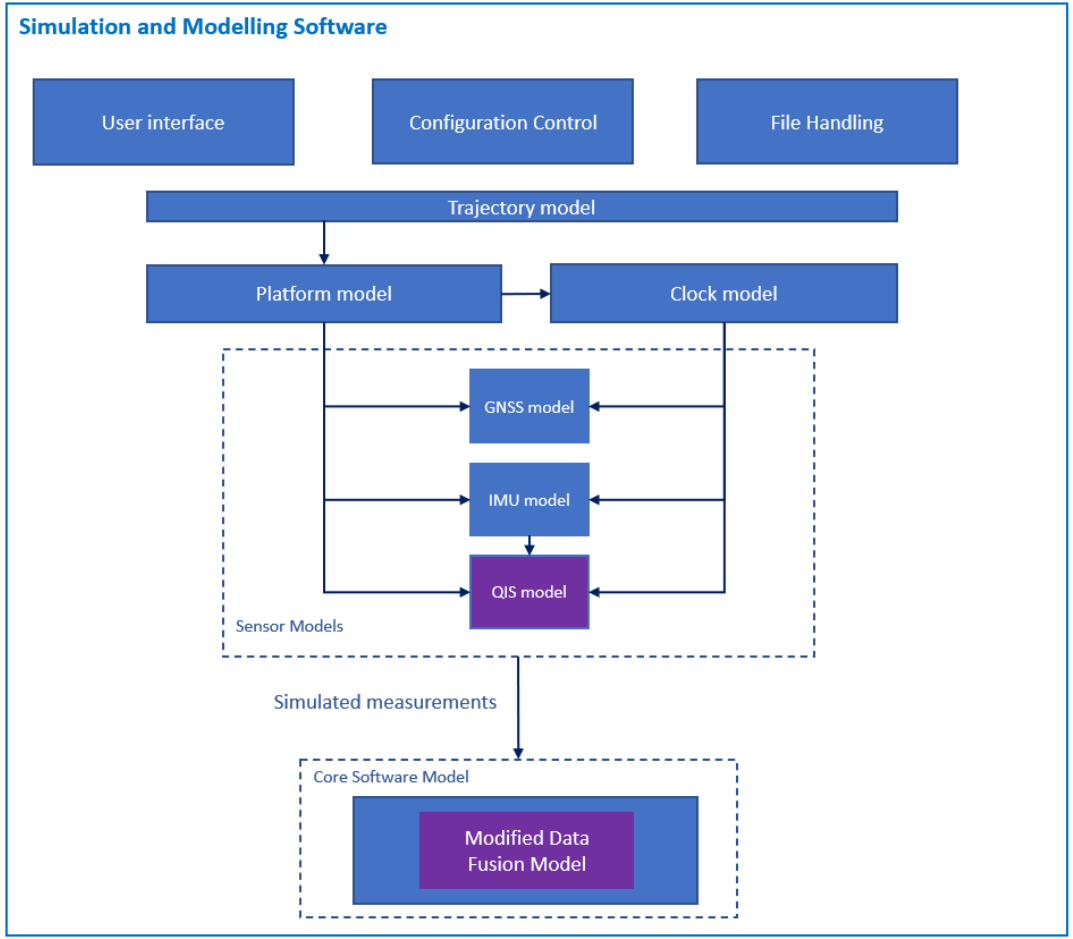
Wang et al., "Improving measurement performance via fusion of classical and quantum accelerometers." 2023

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State of the Art Review: Simulation and Modelling Software

- Trajectory/platform and clock data sent to all sensor models
- Models land vehicle, aircraft, maritime, rail and UAV platforms
- All models highly configurable and customisable
- Motion, vibrations, errors and noise verified against real data to ensure representative of real conditions



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WP2000: Design

Sensor model design

PNT fusion approach

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Quantum Inertial Sensor (QIS) Model Overview

A multi-axis model

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Quantum Inertial Sensor (QIS) Model Overview

A multi-axis model

- Simulates QIS measurements, comprising atomic state population readouts of six CAIs, with two CAIs on each Cartesian axis.
- Atom cloud treated as a point-source which is split into six via three orthogonal ‘cloud-splitter’ pulses (green).
- Three orthogonal sets of counter-propagating Mach-Zehnder (MZ) pulses are then applied (red), separated by interrogation time, T.

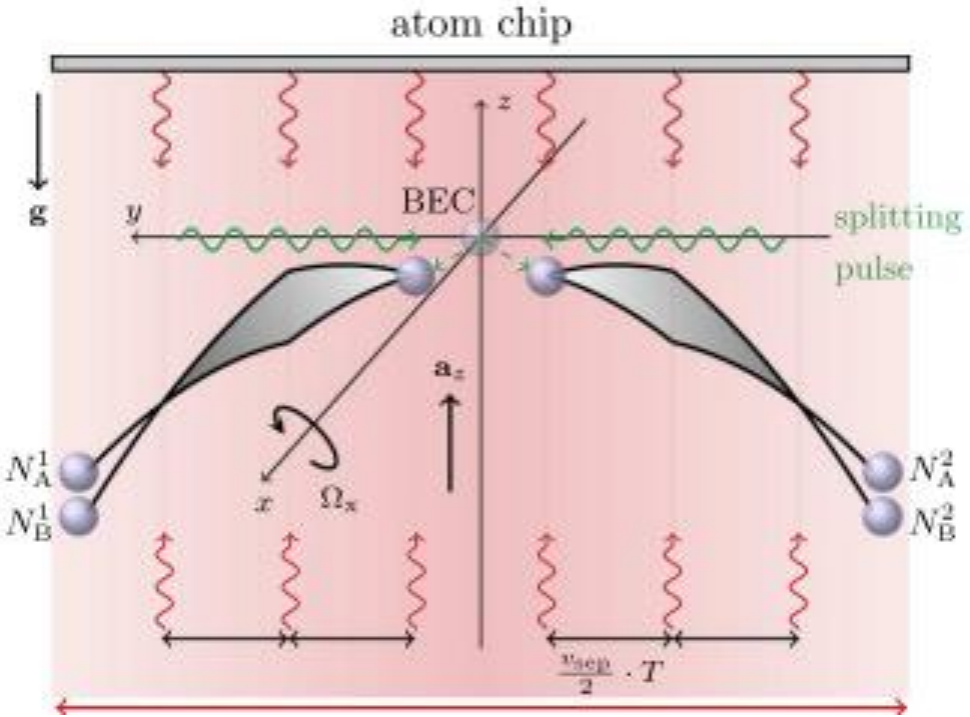
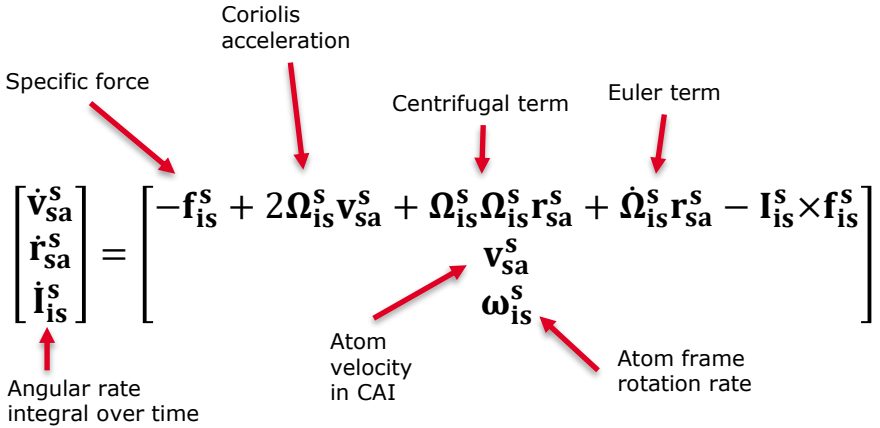


Diagram from Gersemann *et al.,* “Differential interferometry using a Bose-Einstein Condensate,” 2020

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CAI measurement model



Form and solve equations of motion for atoms in each CAI arm

Inertial phase shift

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CAI measurement model

$$\begin{bmatrix} \dot{\mathbf{v}}^s_{sa} \\ \dot{\mathbf{r}}^s_{sa} \\ \dot{\mathbf{I}}^s_{is} \end{bmatrix} = \begin{bmatrix} -\mathbf{f}^s_{is} + 2\boldsymbol{\Omega}^s_{is}\mathbf{v}^s_{sa} + \boldsymbol{\Omega}^s_{is}\boldsymbol{\Omega}^s_{is}\mathbf{r}^s_{sa} + \dot{\boldsymbol{\Omega}}^s_{is}\mathbf{r}^s_{sa} - \mathbf{I}^s_{is}\times\mathbf{f}^s_{is} \\ \mathbf{v}^s_{sa} \\ \boldsymbol{\omega}^s_{is} \end{bmatrix}$$

Form and solve equations of motion for atoms in each CAI arm

Measured population ratio

Contrast

Contrast noise

Inertial phase shift

Laser phase noise

$$P = (P_0 + \delta P_0) - \left(\frac{C + \delta C}{2}\right)\cos(\Phi_I + \Phi_L + \delta\Phi_N + \delta\Phi_L)$$

Measurement offset

Offset noise

Laser phase

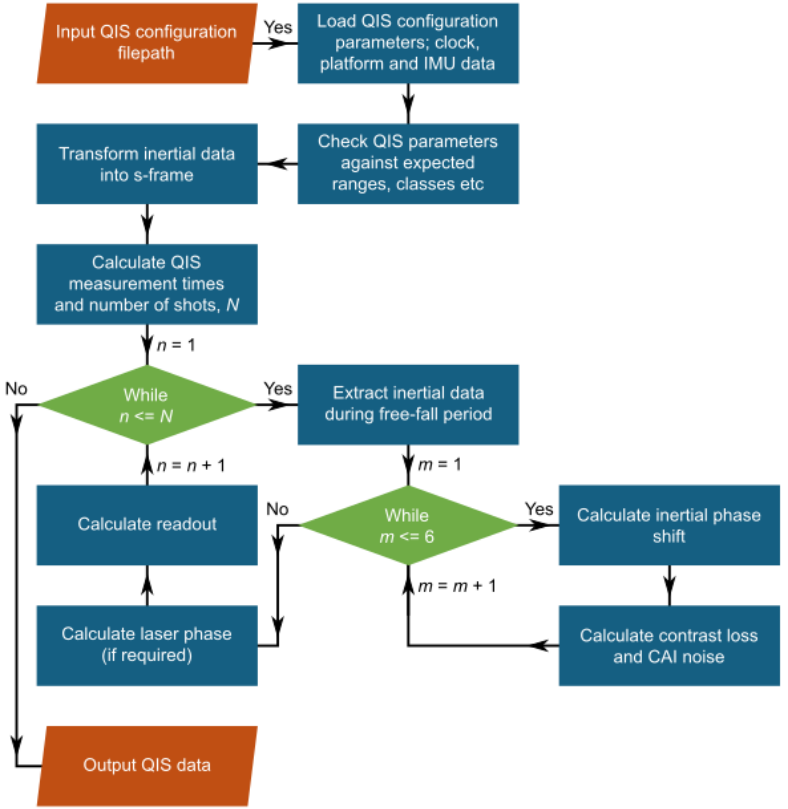
Shot noise

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QIS Model Code Flow

QIS model inputs:
- Data from clock, platform and IMU models
- QIS configuration parameters

QIS model outputs:
- Epochs (defined at end of $2T$)
- CAI readouts (six per epoch)
- Laser phases (six per epoch)



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Atom Trajectories

Atom strapdown equations: Three ODEs which describe the motion of free-falling atoms in each CAI.

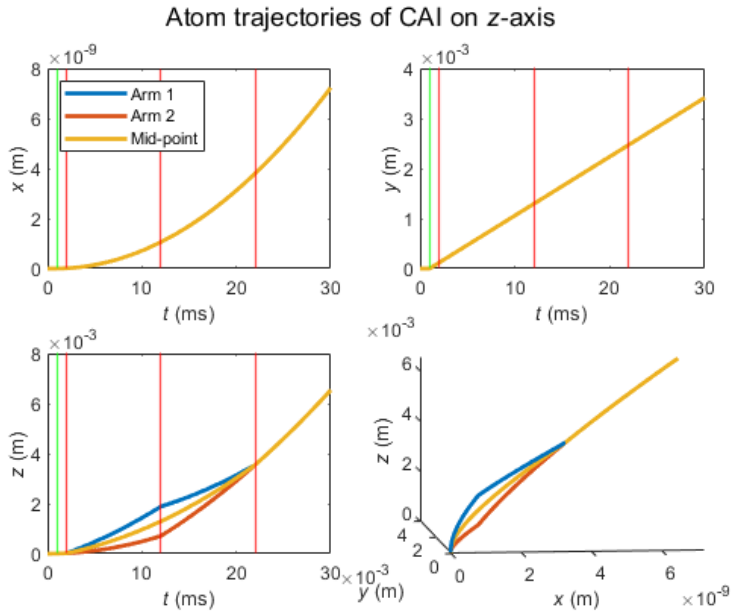
$$\mathbf{Y}(t) = \begin{bmatrix} \mathbf{v}_{\mathbf{sa}}^{\mathbf{s}} \\ \mathbf{r}_{\mathbf{sa}}^{\mathbf{s}} \\ \mathbf{I}_{\mathbf{is}}^{\mathbf{s}} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{\mathbf{sa}}^{\mathbf{s}} \\ \mathbf{r}_{\mathbf{sa}}^{\mathbf{s}} \\ \int_{t_0}^{t} \boldsymbol{\omega}_{\mathbf{is}}^{\mathbf{s}} \, \mathrm{d}\tau \end{bmatrix} \quad \rightarrow \quad \dot{\mathbf{Y}}(t) = \begin{bmatrix} \dot{\mathbf{v}}_{\mathbf{sa}}^{\mathbf{s}} \\ \dot{\mathbf{r}}_{\mathbf{sa}}^{\mathbf{s}} \\ \dot{\mathbf{I}}_{\mathbf{is}}^{\mathbf{s}} \end{bmatrix} = \begin{bmatrix} -\mathbf{f}_{\mathbf{is}}^{\mathbf{s}} + 2\boldsymbol{\Omega}_{\mathbf{is}}^{\mathbf{s}}\mathbf{v}_{\mathbf{sa}}^{\mathbf{s}} + \boldsymbol{\Omega}_{\mathbf{is}}^{\mathbf{s}}\boldsymbol{\Omega}_{\mathbf{is}}^{\mathbf{s}}\mathbf{r}_{\mathbf{sa}}^{\mathbf{s}} + \dot{\boldsymbol{\Omega}}_{\mathbf{is}}^{\mathbf{s}}\mathbf{r}_{\mathbf{sa}}^{\mathbf{s}} - \mathbf{I}_{\mathbf{is}}^{\mathbf{s}} \times \mathbf{f}_{\mathbf{is}}^{\mathbf{s}} \\ \mathbf{v}_{\mathbf{sa}}^{\mathbf{s}} \\ \boldsymbol{\omega}_{\mathbf{is}}^{\mathbf{s}} \end{bmatrix}$$

- Atom trajectories along both CAI arms are calculated numerically using the Runge-Kutta (RK4) solver at pre-defined epochs during the free-fall period.
- Photon recoil from each pulse $i = CS, MZ1, MZ2, MZ3$ is modelled as an instantaneous velocity shift $\mathbf{v}_{\mathbf{rec,i}} = \frac{\hbar}{m}\mathbf{k}_{\mathbf{i}}$.
- Atoms on arm 1 receive 4 kicks $(\mathbf{v}_{\mathbf{rec,CS}}, \mathbf{v}_{\mathbf{rec,MZ1}}, -\mathbf{v}_{\mathbf{rec,MZ2}}, \mathbf{v}_{\mathbf{rec,MZ3}})$.
- Atoms on arm 2 receive 2 kicks $(\mathbf{v}_{\mathbf{rec,CS}}, \mathbf{v}_{\mathbf{rec,MZ2}})$.
- Recoils are applied between numerical integration segments to account for their impact on atom trajectories

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Atom Trajectories

- **For 6 CAIs, this results in 18 trajectories for each QIS measurement:**
  - 6 x arm 1 trajectories, $\mathbf{r}^{\mathbf{s}}_{\mathbf{sa},1}$
  - 6 x arm 2 trajectories, $\mathbf{r}^{\mathbf{s}}_{\mathbf{sa},2}$
  - 6 x mid-point trajectories, $\mathbf{r}^{\mathbf{s}}_{\mathbf{sa,M}}$
- **Figure shows example trajectories for one of the CAIs on the z-axis:**
  - Cloud-splitter pulse kick: $\mathbf{v}_{\mathbf{rec,CS}} = \left[0, \frac{\hbar}{m}|\mathbf{k}_{\mathbf{eff}}|, 0\right]$
  - MZ pulse kicks: $\mathbf{v}_{\mathbf{rec,MZ}} = \left[0,0, \pm\frac{\hbar}{m}|\mathbf{k}_{\mathbf{eff}}|\right]$
  - Note – the axes have a NED orientation.



PNT Fusion Approach

Overview of Multi-Sensor Support

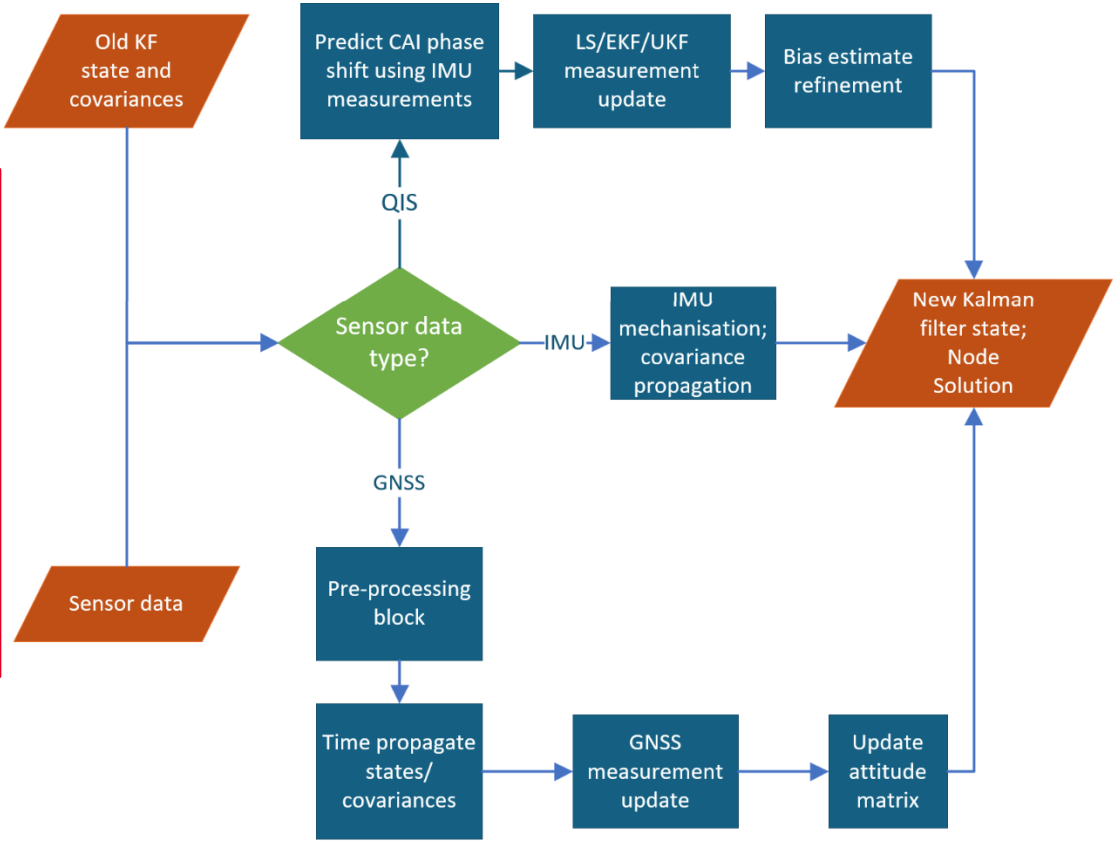
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PNT Fusion Approach

Overview of Multi-Sensor Support

State vector

- ECEF position (x3)
- ECEF velocity (x3)
- Attitude error (x3)
- Accelerometer bias (x3)
- Gyroscope bias (x3)
- GNSS clock state(s)
- GNSS clock drift state(s)



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Quantum Sensor PNT Fusion Approach

Principles of IMU-CAI fusion

- IMU measurements: predict expected CAI phase shift and measurement
- Algorithm compares with actual CAI measurement to allow the biases on the classical IMU to be estimated.

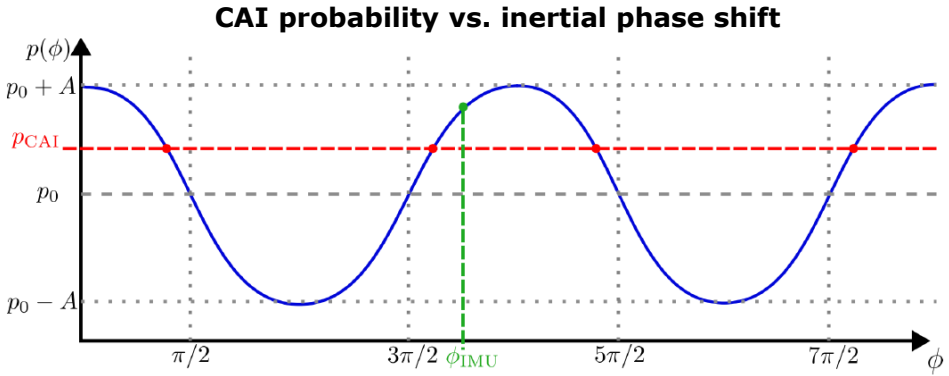
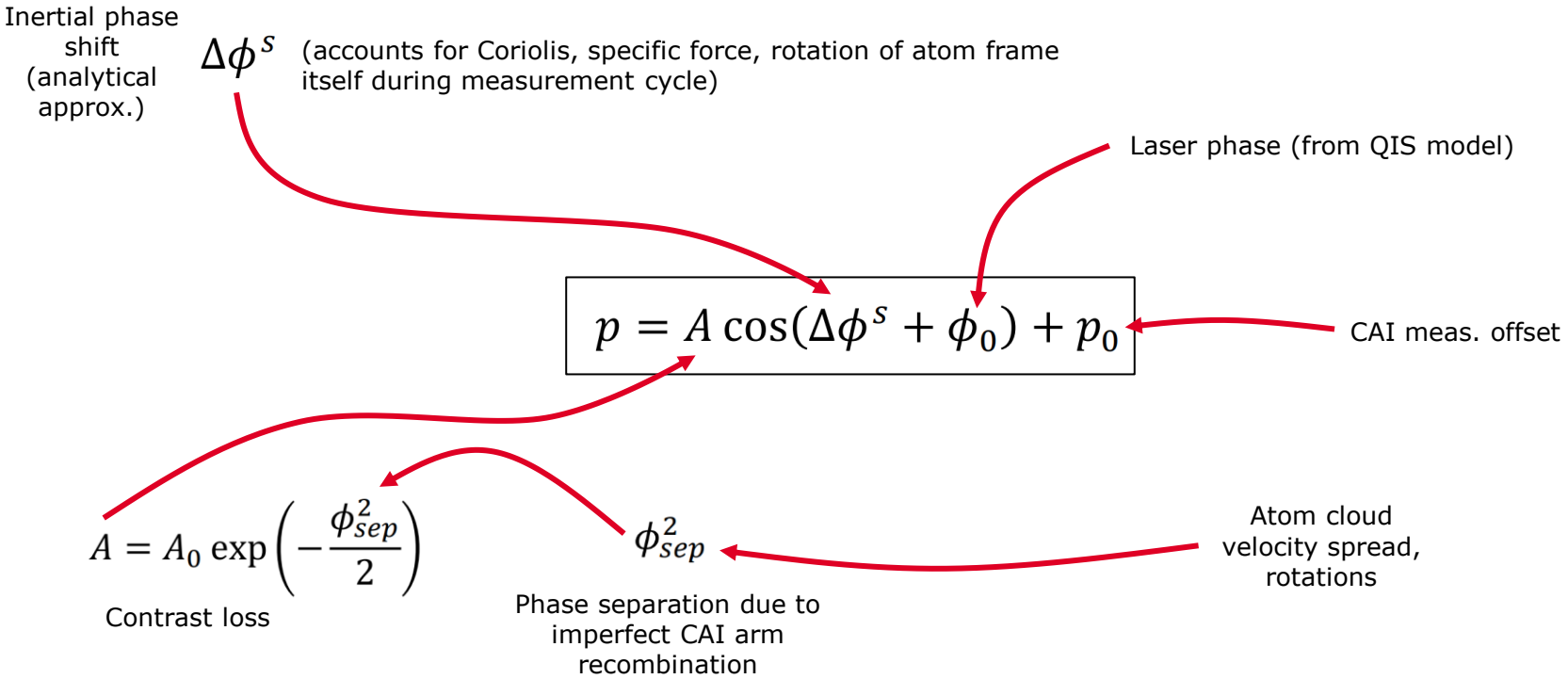


Diagram from Tennstedt, "Concept and Evaluation of a Hybridization Scheme for Atom Interferometers and Inertial Measurement Units," PhD thesis, 2025

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Quantum Sensor PNT Fusion Approach

CAI fusion model



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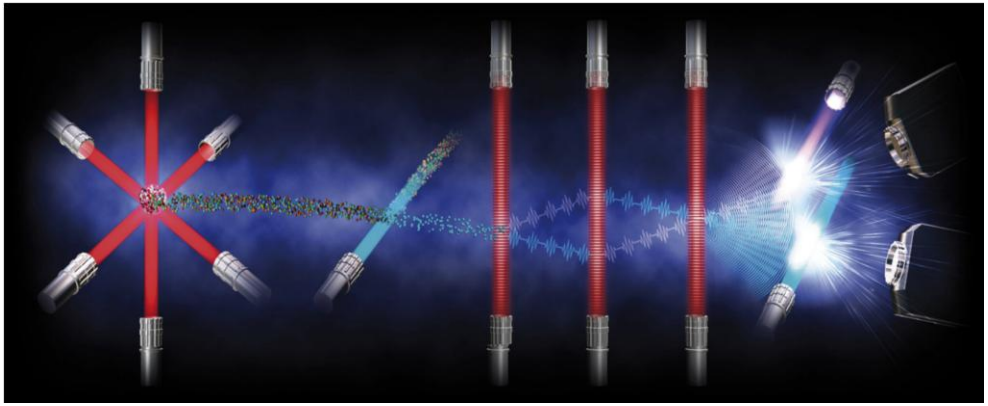
Quantum Sensor PNT Fusion Approach

EKF Measurement update

- Linear region check
- Jacobian matrices
- State update

General observations

- Gravity modelling and attitude error estimation
- "EKF-light"
- Bias refinement



https://www.esa.int/ESA_Multimedia/Images/2022/05/Cold_atom_interferometry_experiment

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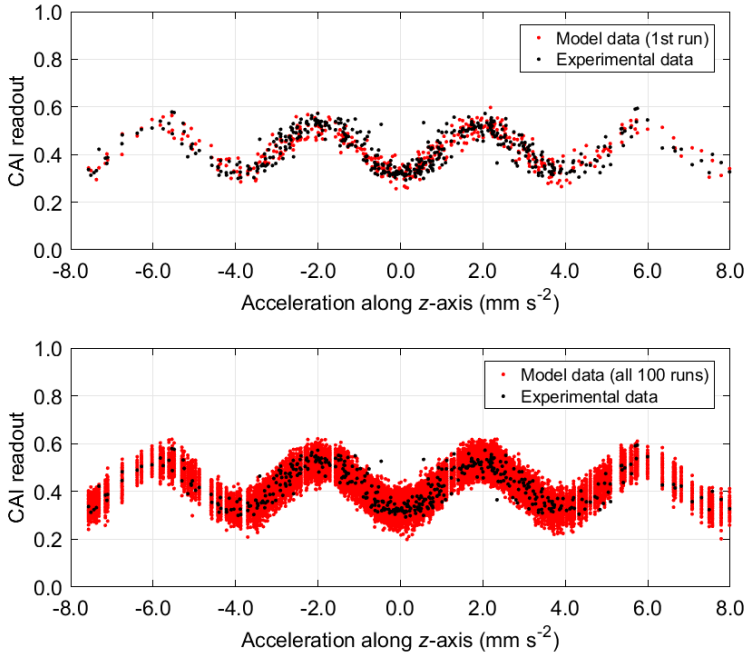
WP3000: Implementation and Testing

Sensor model verification against real data

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QIS Model Verification (Richardson et al 2020)

- T = 10 ms CAI in strapdown.
- Optomechanical resonator attached to mirror.
- Models were configured using the parameter values and data provided in the manuscript.
- Compared experimental and model readout distributions, focusing on reproducing the fringe shape, via Monte-Carlo statistical significance testing.

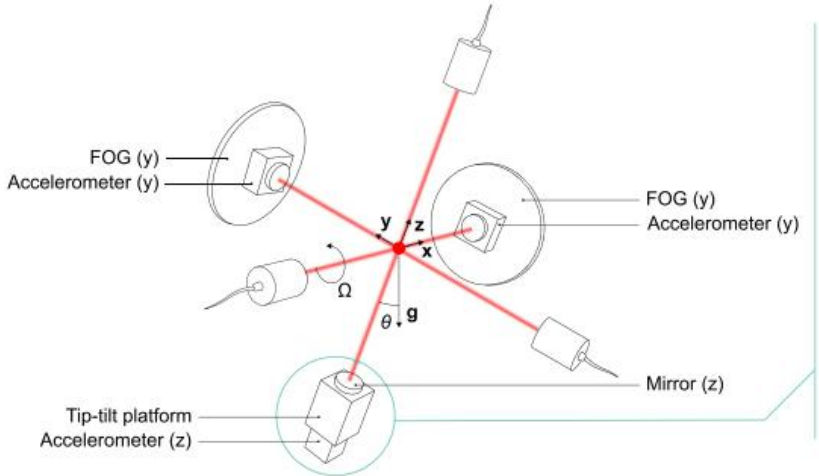


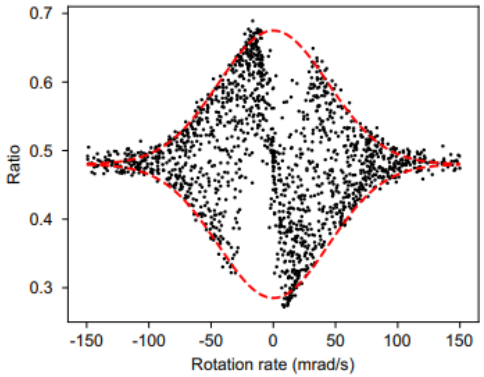
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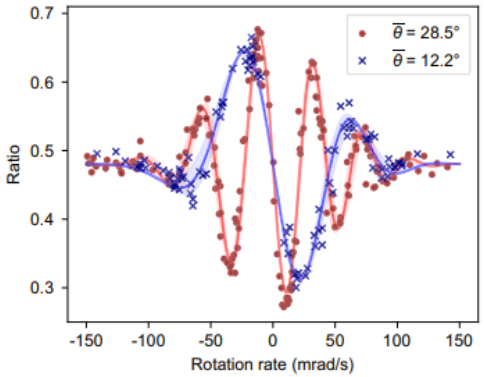
QIS Model Verification (Castanet et al 2024)

- T = 6 ms CAI sensitive to rotations along X-axis in strapdown on rotating platform.
- Accelerometer attached to mirror used to compensate for vibration (but not rotation).

- Models were configured using the parameter values and data provided in the manuscript:
  - **Platform model:** Angular rate and roll tilt along X-axis set to FOG rotation data.
  - **IMU model:** Set to compensate for acceleration along Z-axis.







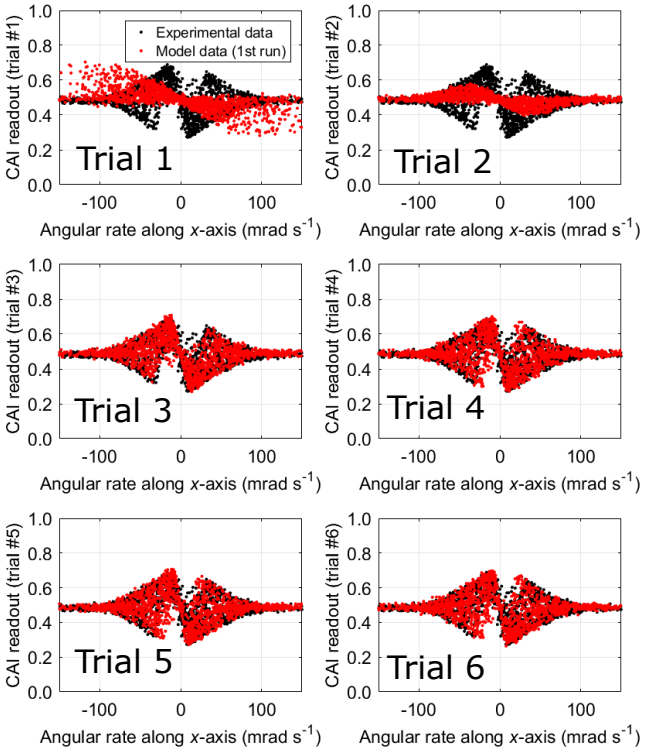
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QIS Model Verification (Castanet et al 2024)

- Compared experimental and model readout distributions, focusing on reproducing the contrast envelope, via Monte-Carlo statistical significance testing.

- To assess impact of different modelling fidelities, the QIS model was run with six different sets of configuration parameters:
  - **Trial 1:** Analytical method; neglect cloud propagation before 1st MZ pulse; neglect deterministic contrast loss.
  - **Trial 2:** Repeat of Trial 1 including deterministic contrast loss.
  - **Trial 3:** Repeat of Trial 2 including cloud propagation before 1st MZ pulse (20 ms).
  - **Trials 4 – 6:** Numerical method using 1st order, 4th order and adaptive time step (ode45) solvers.

- Qualitative agreement best for Trials 3–6.



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WP4000: Experimentation

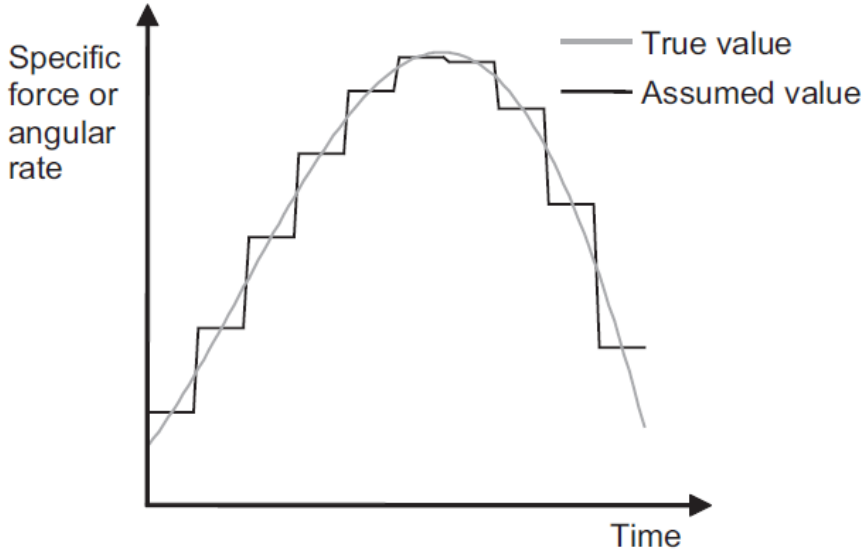
Initial findings

PNT Performance

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Quantum Sensor PNT Fusion

Effect of vibration and averaging errors



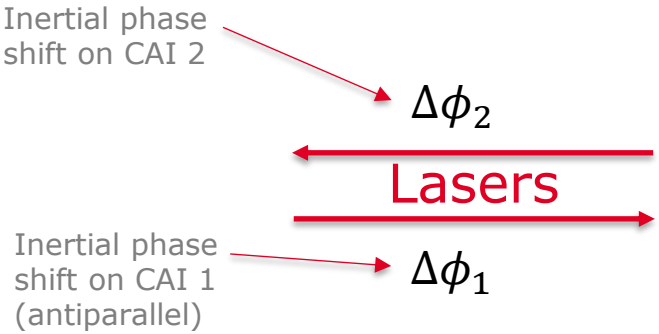
Averaging error of classical IMU.
From P. Groves, *Principles of GNSS, Inertial and Multisensor Integrated Navigation Systems,* 2013

- Platform vibrations exacerbate averaging error (left) inherent in IMU
- Leads to an integer cycle mismatch between IMU-estimated inertial phase and the true phase
- Causes the algorithm to overestimate the bias by many orders of magnitude

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Quantum Sensor PNT Fusion

Effect of vibrations



$\frac{\Delta\phi_1+\Delta\phi_2}{2} \rightarrow$ acceleration

$\frac{\Delta\phi_1-\Delta\phi_2}{2} \rightarrow$ angular rate

→ ***Implementation tips***

- *Keep interrogation time low (1 ms)*
- *Match IMU measurement interval with CAI sensitive interval*
- *High-quality IMU*
- *Try to reduce vibrations at source*

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Simulation Scenarios

- Scenarios were 4 mins long, with 1 min of GNSS outage
- In each case, 3 solutions were traded off:
  1. IMU-QIS-GNSS (QIS estimates biases)
  2. IMU-GNSS (GNSS estimates biases)
  3. IMU-GNSS (no bias estimation)
- Primary interest: performance gain **during GNSS outage**

Platform	Max Speed (m/s)	Max Accel (m/s^2)	Max Turn Rate (deg/s)	Max Climb Rate (m/s)	RMS accel vibration power (m^2/s^4)	RMS ang rate vibration power (deg^2/s^2)	Overall dynamics score
Rail	28	0.3	4	N/A	2.05	0.2	Low
Aircraft	116	5	10	7.3	0.38	0.04	Moderate
Maritime	12.9	1.8	8	N/A	1.36	0.78	High







Images
Rail: https://www.railjournal.com/locomotives/sncf-completes-4km-test-run-of-autonomous-locomotive/
Boat: https://insidegnss.com/gmv-romania-demonstrates-resilient-maritime-pnt-in-the-black-sea/
Aircraft: https://en.wikipedia.org/wiki/Piper_PA-31_Navajo

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Simulation Scenarios: Results

General trend

CAI QIS favours *low dynamics*, *high grade IMUs*

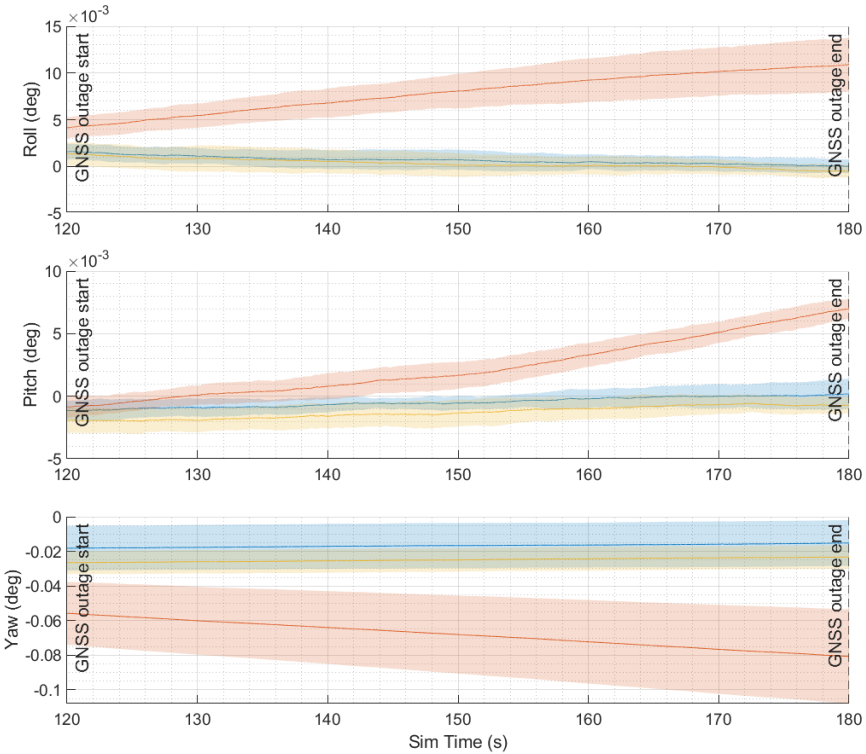
Benefit provided by QIS-derived bias estimates during GNSS outage	High grade IMU	Mid-grade IMU	Low grade IMU
Low Dynamics	Modest	Modest	None
Moderate Dynamics	Modest	Modest	None
High Dynamics	None	None	None

GNSS favours *higher dynamics* and *lower grade IMUs*

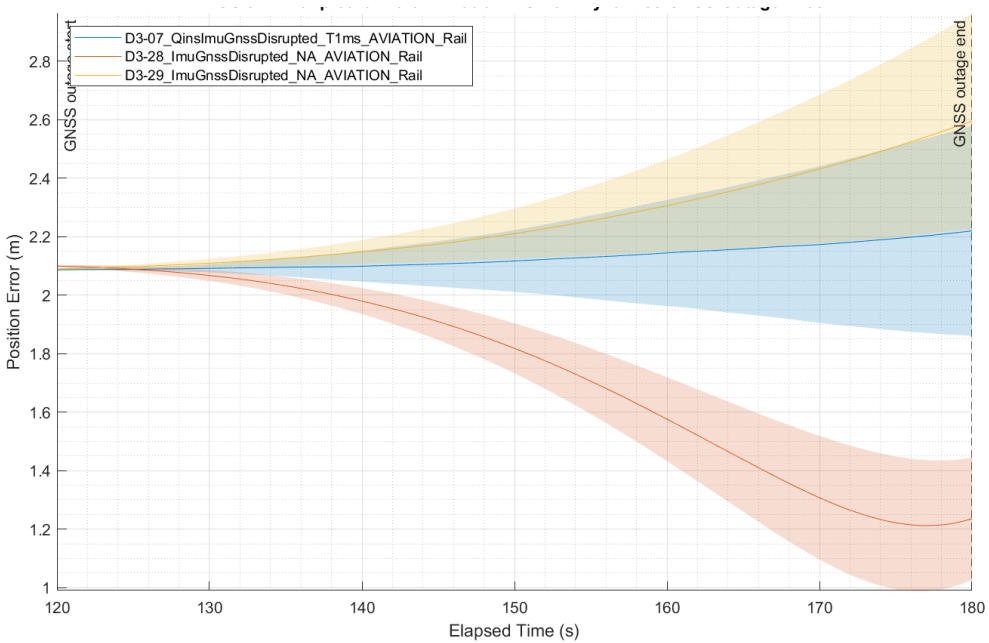
Benefit provided by GNSS-derived bias estimates during GNSS outage	High grade IMU	Mid-grade IMU	Low grade IMU
Low Dynamics	Detrimental	Modest	Large
Moderate Dynamics	None	Large	Large
High Dynamics	Detrimental	Large	Large

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Aviation IMU, low dynamics



Attitude error time series during GNSS outage



3D position error time series during GNSS outage

IMU-QIS

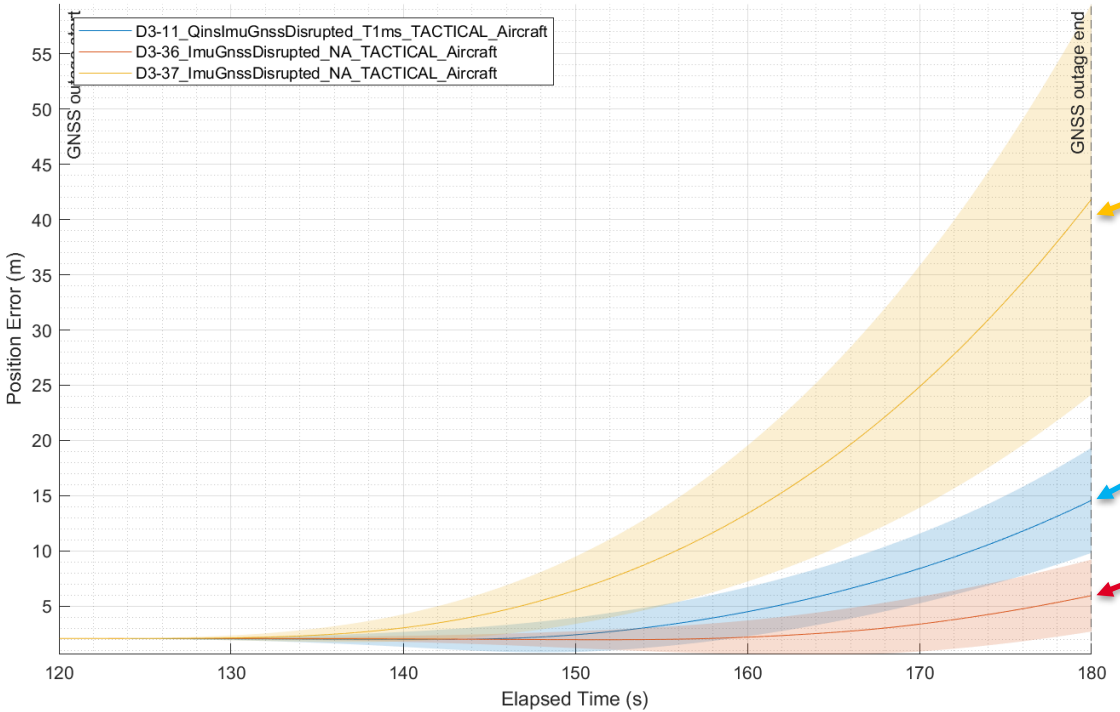
IMU with pre-outage GNSS-derived bias estimates

IMU only

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Tactical IMU, moderate dynamics

Time series plot of 3D position errors during GNSS outage



IMU only

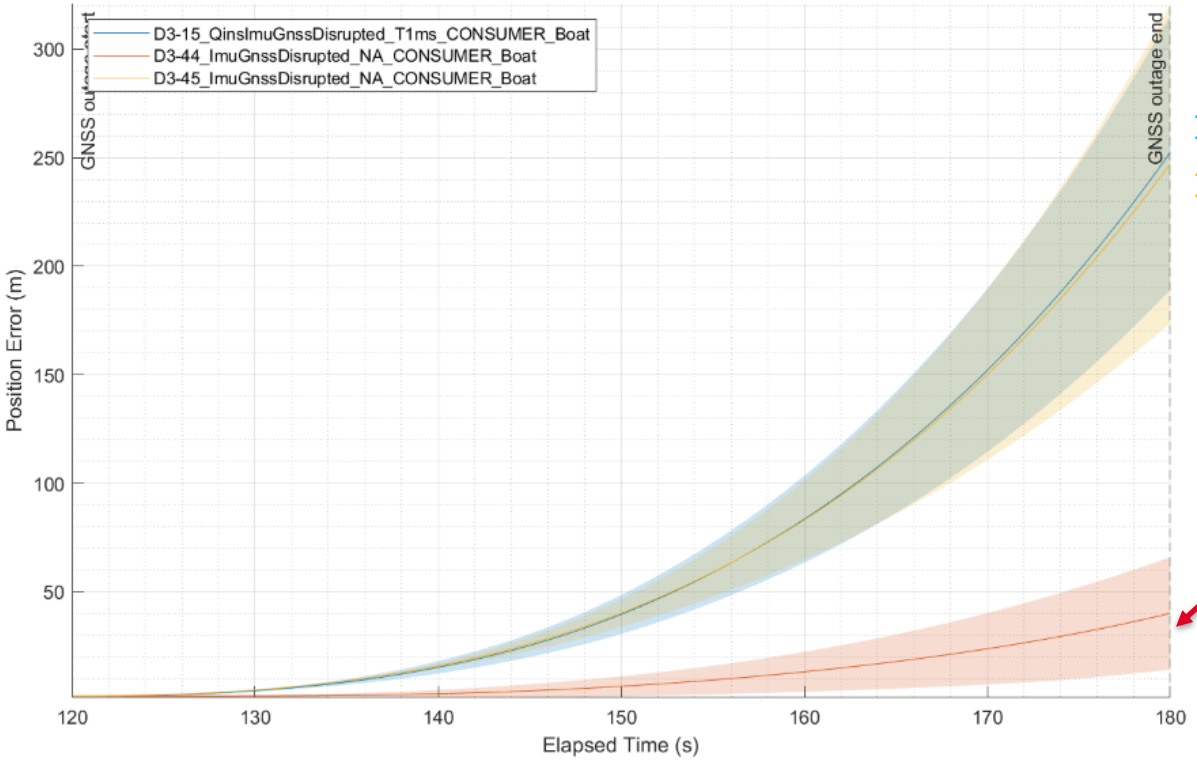
IMU-QIS (continual QIS bias estimation)

IMU corrected by converged GNSS-derived bias estimates from before the outage

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Consumer grade IMU, high dynamics

Time series plot of 3D position errors during GNSS outage



IMU-QIS (continual QIS bias estimation)

IMU only

IMU corrected by converged GNSS-derived bias estimates from before the outage

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WP5000: Conclusions and Recommendations

Technical conclusions

Broader implications

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Conclusions

Technical

- Performance drivers:
  - IMU averaging errors and platform vibrations are dominant error sources
  - Can mitigate if interrogation time T reduced

- PNT solution benefit:
  - Most significant when dynamics are low and IMU grade high
  - Generally greatest for the accelerometer rather than gyro

- Best case
  - Factor of 3-5 improvement pos error
  - Slows error growth but does not remove it

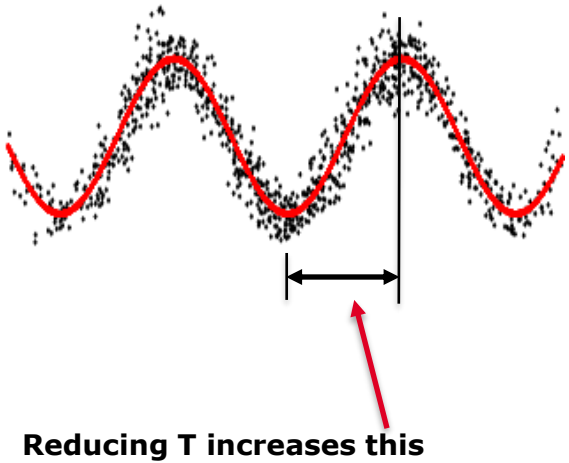


Image from Tennstedt, "Concept and Evaluation of a Hybridization Scheme for Atom Interferometers and Inertial Measurement Units," PhD thesis, 2025

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Implications and Impact

- Key performance drivers identified
- Users interested in QIS – but not just about accuracy.
  Integration, SWaP, reliability, maintainability, cost and practicality also paramount
- Modelling and simulation complements field trials and can identify challenges and demonstrate usability



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Thank you

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